Volatility and Returns: Evidence from China*

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First Draft: January 2019

This Draft: July 2019

Abstract

Long-short factors and industry portfolios in the Chinese A-share stock market tend to have higher returns the months following high volatility. Due to this positive relationship between lagged volatility and returns, volatility-managed portfolios of Moreira and Muir (2017) do not work well in China - they are spanned by the original portfolios. Volatility-scaled portfolios, which increase portfolio exposure in volatile times, are not spanned by the original portfolios and expand the investor's opportunity set. For industry portfolios and long-short factors, the investor's mean-variance frontier shifts towards more desirable regions when volatility-scaled portfolios are added to the investment mix.

* We thank Ram Yamarthy for helpful comments. The views expressed are those of the individual authors and do not necessarily reflect official positions of Paraconic Technologies US Inc.("Paraconic"), and are not necessarily the views of Compass Lexecon ("Compass"), its management, its affiliates, or its other professionals. This article is not an offer to sell or a solicitation of an offer to buy any investment product or services offered by Paraconic or Compass. Paraconic and Compass do not guarantee the accuracy or completeness of the information contained herein, and any information provided by third parties has not been independently verified by Paraconic or Compass. All errors are our own.

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1. Introduction

There is strong theoretical basis to believe risk and return are positively related. Risk-averse investors value higher returns and lower volatility, so risky investments must offer higher returns in equilibrium. In his groundbreaking work on Modern Portfolio Theory, Markowitz (1952, 1959) demonstrates how investors can quantify their risk-return tradeoff by measuring portfolio expected returns against portfolio volatility. Since Markowitz, asset pricing theory and empirics have been built around measuring and testing various forms of risk-return tradeoffs. Workhorse asset pricing models often imply that in equilibrium, investors must take on additional risk if they want higher returns.

The empirical evidence between risk and return is less clear. In its most basic form, a positive risk-return tradeoff implies higher volatility is associated with higher returns. Although there is a large literature on this topic, evidence of a positive risk-return tradeoff has been mixed. Campbell and Hentschel (1992) and French et al. (1987) find a positive relationship between conditional expected returns and conditional variance, whereas Campbell (1987) and Glosten et al. (1993) find a negative relationship. Contradictory empirical results may be partially attributed to different research designs, but may also reflect a weak relationship buried in noisy data. Although empirical findings have been mixed, it is clear that empirical research on the relationship between volatility and returns is of central importance in financial economics. Whereas much of the existing literature focuses on the U.S. markets, we turn our attention to the Chinese A-share stock market.

Established in 1991, China's stock market has gone through rapid development. It has become the second largest stock market in the world with a market capitalization of $5 trillion by August 2016 (Chen and Chi, 2018). While the Chinese stock market shares some similar characteristics as other large economies (Carpenter et al., 2015), it does have its unique institutional features. For example, according to official statistics from both the Shanghai and Shenzhen stock exchanges, more than 80% of the trading volume can be attributed to retail investors. In contrast, institutional investors dominate trading in the U.S. stock market. As retail and institutional investors have different goals and behave differently, asset prices can be impacted in different ways in a retail-dominated market compared to an institution-dominated market.

This paper investigates the empirical relationship between volatility measures and returns for Chinese A Shares. We document a key empirical fact about volatility and returns: there is a positive relationship between lagged volatility and future returns. Figure 1 illustrates this positive risk-return tradeoff for the A Shares value-weight market portfolio.

[Figure 1 about here]

Figure 1 stands in sharp contrast to Figure 1 of Moreira and Muir (2017), in which expected returns show a lack of variation across the five volatility buckets for the U.S. value-weight market. For the Chinese A Shares, higher volatility appears to be associated with higher future returns. We also find that lagged change in volatility to be positively associated with future returns. These patterns hold for market returns, Fama and French (1992) factors, momentum (Carhart, 1997), and 63 industry portfolios defined by Global Industry Classification Standard (GICS).

The relationship between contemporaneous volatility measures and returns is less striking. As the case for the U.S. market, there is a weak and negative relationship between contemporaneous volatility and returns, and a negative relationship between change in volatility and same period returns (Campbell, 1987; Glosten et al., 1993).

A positive relationship between lagged volatility and returns has important implications for using volatility as a portfolio management tool. Moreira and Muir (2017) demonstrate that for the U.S. equity markets, scaling portfolio returns inversely proportional to lagged variance produces large alphas and higher Sharpe ratios compared to the original portfolios. Because volatility positively forecasts returns in Chinese A Shares, the Moreira and Muir (2017) approach is not suitable for China; managing the portfolio exposure to be inversely proportional to lagged variance ignores the positive predictive power of lagged volatility. Volatility-managed portfolios are spanned by the original portfolios: time series regressions of volatility-managed portfolios on the original portfolios have negative intercepts. Volatility-managed portfolios do not help the investor improve her investment opportunity set.

We propose an alternative construction, volatility-scaled portfolios, which increases portfolio exposure when volatility is high and decreases portfolio exposure when volatility is low. This portfolio management technique takes advantage of the positive relationship between lagged volatility and future returns observed in Chinese A-share market. In spanning regressions of volatility-scaled portfolios on the original portfolios, the intercepts are economically large and range between 1% to 6% per year, indicating that volatility-scaled portfolios are not spanned by the original portfolios and can expand the investor's opportunity set. Our results are unchanged if we use GARCH volatility forecasts, rather than lagged volatility, to adjust portfolio exposure.

We also quantify the change in the investor's opportunity set by comparing the investor's mean-variance frontier before and after adding volatility-scaled portfolios. The mean-variance frontier including volatility-scaled portfolios subsumes the mean-variance frontier excluding them, moving the investor's feasible set towards higher-return and lower-volatility portfolios. As higher Sharpe ratio portfolio combinations become available, the investor's expanded choice set can improve his risk-return tradeoff. Overall, the investor is better off adding volatility-scaled portfolios to his investment mix.

We consider the predictions of several leading asset pricing models including habit formation of Campbell and Cochrane (1999), disaster risk of Wachter (2013), long-run risk of Bansal and Yaron (2003), and the intermediary-based model of He and Krishnamurthy (2013). Moreira and Muir (2017) show that these models imply zero or negative intercepts in spanning regressions of volatility-managed portfolios onto the original portfolios, which they reject for the U.S. markets. For the Chinese A-share market, indeed we find zero or negative intercepts for volatility-managed portfolios, consistent with the leading asset pricing models.

Our paper most directly fits into the literature on risk-return tradeoffs in Chinese A Shares. Kong et al. (2008) uses MIDAS, a mixed-frequency technique, and finds no relationship between volatility and returns for the aggregate market from 1993 to 2001. However, they find a positive tradeoff for 2001 to 2005. Chen (2015) adopts a GARCH-M specification to find a positive risk-return relationship for the Shenzhen Stock Exchange but not for the Shanghai Stock Exchange. Lee et al. (2001) also uses GARCH-M but does not find any relation between expected returns and risk. Compared to these papers, we make use to other stock market factors and industry portfolios to study risk-return tradeoffs beyond the market portfolio. We also explore using volatility in portfolio management.

More broadly, our paper is related to the empirical literature documenting risk-return relationships. Much of the existing work focuses on the U.S. markets. French et al. (1987) apply a GARCH-M model to find a positive relationship between expected risk premiums and volatility. Campbell and Hentschel (1992) find evidence of "volatility feedback" using a model that combines GARCH and the Campbell and Shiller (1988a, b) identity. Campbell (1987) finds a negative relationship between conditional variance and stock returns using a variety of linear models. Glosten et al. (1993) use a modified GARCH-M model and find a negative relation between conditional mean and variance for monthly returns. Compared to these studies, our paper takes a simple and straightforward approach of looking at the relationship between realized volatility and returns. For the Chinese stock market, clear empirical patterns emerge without resorting to sophisticated statistical techniques.

Applying volatility measures in portfolio management is a central theme of this paper. Moreira and Muir (2017) document that scaling portfolios by the inverse of their lagged variance improves the performance of the original portfolios. Qiao et al. (2018) show that scaling portfolios using the inverse of their downside variance further improves the investor's opportunity set. In contrast to these approaches, our paper demonstrates that in Chinese A Shares, cutting portfolio exposure when volatility is high is suboptimal, because high volatility is associated with higher returns next period. Our alternative portfolio management tool, volatility-scaled portfolios, accounts for the unique stylized fact in Chinese A Shares and helps expand the investor's opportunity set.

Our paper also relates to the literature on time-varying discount rates. In his presidential address to the American Finance Association, Cochrane (2011) asserts that the current organizing idea in empirical asset pricing is time-varying discount rates. Discount rates vary over time and are predictable using the proper information set. Early work by Fama and French (1989) and Campbell and Shiller (1988a, 1988b) convincingly demonstrate the time-varying nature of discount rates. In our attempt to understand the relationship between volatility and returns, we uncover new empirical findings on time-varying discount rates for the Chinese stock market: discount rates vary over time and past volatility captures some expected return variation.

The paper is organized as follows. Section 2 presents the relationship between volatility measures and returns and documents that lagged volatility and change in volatility are positively correlated with future returns. Section 3 investigates the portfolio management implications of time-varying volatility and returns, including analysis on volatility-managed portfolios, volatility-scaled portfolios, and changes in the mean-variance frontier. Section 4 discusses our findings. Section 5 concludes.

2. The Relationship between Returns and Volatility

2.1 Data

We collect Chinese A-share market data from WIND®. We cover all publicly listed stocks in Shanghai Stock Exchange and Shenzhen Stock Exchange, which comprises of 2,891 stocks as of December 2015. This is also the stock universe for the Chinese mutual funds in our. Our dataset includes daily data of stock returns, trading status, market capitalization, high, low, open, close, value-weighted average price, and major index returns (SSE50, CSI300, and CSI500), annual data of book value at the end of each June, industry classifications following GICS, and IPO dates.

We construct the common stock return factors using the Chinese stock data. The market risk premium, Rm-Rf, is taken as the value-weight one-month return on publicly-listed A-share stocks on the Shanghai and Shenzhen exchanges. Weights are monthly market capitalizations. Rf is the risk-free rate captured by the three-month Chinese household deposit rate. This rate is reported as an annual figure, so we divide by 12 to get a monthly Rf.

We construct the size and value factors using a similar procedure as Fama and French (1992). Each stock is categorized as "big" or "small" based on whether it is above or below the median float market cap at the end of June each year. The book-to-market ratio, BM, is calculated as the shareholder's equity (less minority equity) divided by the total market cap. The book value comes from the last available financial report that has been released on the appointed day, and the market value comes from the data at month end. Stocks are classified as "high", "medium", or "low" based on the BM ratio at the end of June for each stock. Stocks with top 30% BM are classified as "high". The bottom 30% is classified as "low". The 30th to 70th percentile BM stocks are classified as "medium". Six portfolios are formed using market cap and BM breakpoints: Small/High, Small/Medium, Small/Low, Big/High, Big/Medium, and Big/Low.

In addition to annual assignment of stocks into the six portfolios, we also consider monthly assignment. The market cap breakpoints each month is the median market capitalization at the end of last month. For BM breakpoints, stocks are classified as “high”, “medium”, or “low” depending on its BM ratio at the end of last month. We then form six portfolios using the market cap and BM breakpoints, as the annual sort, at the end of each month.

Within each of the six portfolios, value-weight monthly returns are computed. The size factor, SMB, is the equal-weight average of Small/High, Small/Medium, and Small/Low portfolios minus the equal-weight average of Big/High, Big/Medium, and Big/Low portfolios. HML is constructed as the equal-weight average of Small/High and Big/High minus the equal-weight average of Small/Low and Big/Low.

We compute past returns from 12 months prior to two months prior for each stock. The breakpoints for past performance are the 30th and 70th percentiles. The bottom 30% past performers are classified as “losers”; the top 30% of past performers are classified as “winners”. The middling performers are classified as “neutral”. Each month, we form six portfolios combining past performance and market capitalization: Small/Loser, Small/Neutral, Small/Winner, Big/Loser, Big/Neutral, and Big/Winner. The momentum factor, MOM, is constructed as the equal-weight average of Small/Winner and Big/Winner minus the equal-weight average of Small/Loser and Big/Loser. We present summary statistics for the factors in Table 1.

[Table 1 about here]

Industry portfolios are value-weight within each GICS classification. We omit Real Estate Management & Development, GICS code 601020, because it only contains 13 months of returns. We focus on 63 industry portfolios for our analysis.

2.2 Contemporaneous Volatility Measures and Returns

We look at the contemporaneous relationship between volatility and returns by regressing monthly returns onto the same month realized volatility:

$$f_t = a + b\sigma_t + \eta_t \quad (1)$$

where f_t is the return at time *t* for long-short factors or long-only industry portfolios. σ_t is the month *t* standard deviation of f_t constructed using daily observations. b is the coefficient of interest which measures the comovement between returns and volatility. a is a constant and η_t is the time *t* residual. The regression coefficient b and the associated t-statistics for long-short factors are shown in Table 2. Results for industry portfolios are shown in Figure 2.

[Table 2 about here]

The contemporaneous relationship between returns and volatility is weak for factors. Market, SMB_Annual, HML_Annual, and SMB_Monthly all show statistically insignificant coefficients. For the two factors that have significant coefficients, MOM shows a negative relationship between volatility and same period returns, whereas HML_Monthly shows a positive relationship. The pooled regression coefficient is -0.03 with a t-statistic, clustered by portfolio and by time, of -0.8.

[Figure 2 about here]

We look across 63 industry portfolios in Figure 2. Some portfolios show a small positive relationship between returns and volatility in the same period, whereas other portfolios show a small negative relationship. There are 32 industry portfolios with positive coefficients and 31 portfolios with negative coefficients. The pooled regression coefficient is economically and statistically small. As is the case for the U.S. equity markets, the contemporaneous relationship between volatility and returns in Chinese A Shares is weak.

A natural next step is to uncover the relationship between volatility innovations and returns. For the U.S. equity markets, there is a pronounced “leverage effect”: negative returns are associated with larger increases in volatility than positive returns of the same magnitude (Campbell and Hentschel, 1992; Glosten et al., 1993). Unconditionally, this effect leads to a negative correlation between volatility innovations and returns. We examine the relationship between volatility innovations and returns in the Chinese A-share market, using monthly change in volatility as a measure of volatility innovations. Our regression specification is as follows:

$$f_t = a^{ch} + b^{ch}\Delta\sigma_t + \eta_t^{ch} \quad (2)$$

where $\Delta\sigma_t = \sigma_t - \sigma_{t-1}$ is the first difference in monthly realized volatility. a^{ch} and η_t^{ch} are the intercept coefficient and regression residual. b^{ch} is the regression coefficient measuring the relationship between portfolio returns f_t and $\Delta\sigma_t$.

We present the factor results in the lower panel of Table 2 and industry results in Figure 3. There is still considerable variation across the factors, with three factors showing positive coefficients and three showing negative coefficients. The pooled regression shows a statistically significant coefficient of –0.10 across six factors.

The picture is clearer for industry portfolios. In Figure 3, many regression coefficients are negative and economically large, ranging between -0.1 and -0.4. 50 of 63 industries display a negative relationship between returns and the change in volatility; 13 industries show a positive relationship. Furthermore, the t-statistics are larger compared to those in Figure 2; many of the negative coefficients are statistically significant at the 5% level. The pooled regression coefficient is statistically significant at the 1% level.

[Figure 3 about here]

Overall, the relationship between returns and contemporaneous volatility is weak, whereas the relationship between returns and contemporaneous volatility innovations is stronger and typically negative. These results are similar to those from the U.S. markets. In the next section, we examine the relationship between returns and lagged volatility measures, where we discover important differences to the U.S. markets.

2.3 Lagged Volatility Measures and Returns

We explore the relationship between returns and lagged volatility. There is a large literature exploring the ability of volatility to forecast futures returns. The prevailing empirical finding for the U.S. market is that there is limited, if any, relationship between past volatility of a portfolio and future portfolio returns. This result is a main motivating fact for Moreira and Muir (2017), who note that by scaling portfolios (including the market portfolio) by their past volatilities improves the risk-return properties of those portfolios.

We measure the relationship between lagged volatility and current returns through forecasting regressions:

$$f_t = \gamma + \delta\sigma_{t-1} + \varepsilon_t \quad (3)$$

Where f_t is the portfolio return at time t. σ_{t-1} is the standard deviation of f_{t-1} constructed using daily observations. γ is the intercept and ε_t is the residual. δ is the forecasting coefficient. A positive δ indicates that higher volatility in the previous month is associated with higher returns this month.

The top panel of Table 3 presents the forecasting coefficients and the associated t-statistics for factor portfolios. Four of the fix factors show positive coefficient, whereas the pooled regression coefficient is 0.04 and not statistically significant. Figure 4 presents the forecasting coefficient δ and t-statistics for industry portfolios. For most of the return series, last month's volatility has some predictive power for this month's returns. We see a positive risk-return tradeoff: If last month's volatility was high, this month's return is likely to be higher. Among 63 industry portfolios, only two have marginally negative δ. The pooled coefficient for industry portfolios has a large t-statistic of 6.8; we reliably estimate a positive relationship between return and lagged volatility for industry portfolios.

[Table 3 about here]

The results in Figure 4 stand in sharp contrast to the case for the U.S. equity markets. For long-short factors or long-only industry portfolios in the U.S. markets, past month's volatility does not forecast this month's returns for any of these series.

[Figure 4 about here]

We also explore forecasting regressions using the volatility innovations as measured by the first difference:

$$f_t = \gamma^{ch} + \delta^{ch}\Delta\sigma_{t-1} + \varepsilon_t^{ch} \quad (4)$$

Where $\Delta\sigma_{t-1} = \sigma_{t-1} - \sigma_{t-2}$ is the lagged first difference in monthly realized volatility. δ^{ch} is the forecast coefficient of interest. The bottom panel of Table 3 and Figure 5 show the results of these forecasting regressions.

There is a positive but economically small relationship between returns and lagged volatility innovations: five of six factors show positive coefficients, and the pooled coefficient is positive. However, the magnitude of the coefficients is generally small. Industry portfolios show stronger results in Figure 5. 55 of 63 industry portfolios show a positive coefficient; 8 are negative. The pooled coefficient is 0.05 with a t-statistic of 6.1, demonstrating a reliably positive relationship between lagged volatility innovations and returns for industry portfolios. The change in month-over-month volatility has positive predictive power for the next month's returns. If volatility increased last month, returns are likely to be higher this month.

[Figure 5 about here]

The Chinese A-share market exhibit unique and intriguing patterns for the relationship between volatility and returns. There exists a weak or negative relationship between returns and contemporaneous volatility - qualitatively similar to the U.S. equity markets. However, the positive relationship between lagged volatility or change in volatility and returns appear to be unique to the Chinese equity market. For the U.S. markets, there is no obvious relationship between lagged volatility and current period returns. This difference in risk-return tradeoff between the U.S. and Chinese stock markets has important implications for using volatility for portfolio management. We explore these implications in the following section.

3. Volatility in Portfolio Management

3.1 Volatility-Managed Portfolios

Moreira and Muir (2017), exploiting the empirical fact that lagged volatility and current period returns are not closely linked, propose managing portfolio exposures through scaling return series by the inverse of their lagged variance:

$$f_{t+1}^{MM} = \frac{c}{\sigma_t^2} f_{t+1} \quad (5)$$

Where f_{t+1}^{MM} is the Moreira and Muir (2017) volatility-managed portfolio. f_{t+1} is the original portfolio. σ_t^2 is the variance estimated using last month's daily observations. c is a constant set such that f_{t+1}^{MM} and f_{t+1} have the same unconditional standard deviation. Moreira and Muir (2017) show that their volatility-managed portfolios are able to expand the investor's opportunity set.

We investigate the benefits of Moreira and Muir's (2017) portfolio construction in the Chinese A-share market. We first construct volatility-managed factors using the methodology from Moreira and Muir (2017), then we run the following regression to see if f_{t+1}^{MM} expands the investment opportunity set:

$$f_{t+1}^{MM} = \alpha + \beta f_{t+1} + \varepsilon_{t+1}^{MM} \quad (6)$$

The purpose of Equation (6) is to measure whether the volatility-managed portfolio f_{t+1}^{MM} can expand the investor's opportunity set relative to the original portfolio returns f_{t+1}. The coefficient of interest is α: if it is large and positive, f_{t+1} cannot span f_{t+1}^{MM}. Adding f_{t+1}^{MM} to the investor's set of investments broadens his investment opportunities and expands his mean-variance frontier.

We present the estimated intercept α and the associated t-statistics in Figure 6. Across long-short factors and long-only industry portfolios, the majority of estimated intercepts is negative. 60 of 69 return series exhibit negative intercepts. The largest positive estimated intercept is for MOM, 7.9%. In comparison, 19 industry portfolios show negative intercepts of -10% or greater.

[Figure 6 about here]

Scaling returns series by the inverse of past month's variance does not expand the investor's opportunity set relative to the original portfolio. This finding stands in contrast to the results in Moreira and Muir (2017) for the U.S. equity markets, for which the authors find that volatility-managed portfolios are almost never spanned by the original portfolios, and the intercept estimates are economically and statistically large.

Our results differ from those from Moreira and Muir (2017) because there is a positive risk-return tradeoff between lagged volatility and returns. If last month's volatility (or variance) is high, this month's returns are likely to be higher for the majority of our test portfolios. Managing portfolio exposure to be proportional to the inverse of past month's variance ignores this return predictability and potentially weakens the portfolio risk-return tradeoff. Because of this empirical fact unique to the Chinese A-share market, Moreira and Muir (2017) volatility-managed portfolios do not benefit A Shares investors.

The big outlier in Figure 6 is momentum, for which the volatility-managed version is not spanned by the original factor, and in fact has a statistically large intercept estimate of 7.9% per year. In Table 3, for the momentum factor, past volatility and the change in past volatility showed negative predictive coefficients for next month's returns. Therefore, managing portfolio exposure to be proportional to the inverse of past month's variance may improve the risk-return tradeoff of momentum returns through exploiting this negative predictive relationship.

If volatility-managed portfolios do not expand the investor's opportunity set for A Shares, can volatility still be useful for portfolio management? We exploit the positive return predictability of lagged volatility, and we propose an alternative portfolio management methodology that does expand the investor's opportunity set for Chinese A Shares.

3.2 Volatility-Scaled Portfolios

In Chinese A Shares, higher volatility in one month is associated with higher returns in the following month. Rather than managing portfolio exposure to be inversely proportional to lagged variance, we consider volatility-scaled portfolios that increase position when volatility is high and decrease position when volatility is low. Such portfolio construction takes advantage of the positive association between lagged volatility and future returns. Volatility-scaled portfolio f_{t+1}^{σ} for return series f_{t+1} is constructed as follows:

$$f_{t+1}^{\sigma} = \frac{\sigma_t}{k} f_{t+1} \quad (7)$$

Where σ_t is the standard deviation estimated using daily observations of f_t. k is a constant such that f_{t+1}^{σ} and f_{t+1} have the same unconditional standard deviation.

We conduct spanning tests for volatility-scaled portfolios, similar to those for volatility-managed portfolios:

$$f_{t+1}^{\sigma} = \alpha^{\sigma} + \beta^{\sigma} f_{t+1} + \varepsilon_{t+1}^{\sigma} \quad (8)$$

The spanning regression results are shown in Figure 7. Intercept estimates are generally positive and economically large. 32 of 69 return series have intercepts that range from 1% to 6% per year; 45 series have positive intercepts. Momentum and the industry portfolio 351020, "Health Care Providers & Services", are two portfolios that have economically large negative intercept estimates, indicating the volatility-scaled portfolios of these two return series are spanned by the original return series. Momentum has the only statistically large negative intercept. Overall, scaling return series proportionally to lagged volatility appears to improve the investor's opportunity set.

[Figure 7 about here]

Although the economic magnitudes of the intercepts in Figure 7 are large, they are generally not statistically significant. We do not have enough power to reject the null of zero intercept for most of the spanning regressions. To increase the power of our test and demonstrate the expansion of the investor's opportunity set, we show how the investor's mean-variance frontier changes when we include volatility-scaled portfolios in the next section.

3.3 Investor's Mean-Variance Opportunity Set

The previous section provides univariate comparisons of volatility-scaled portfolios and the original portfolios. We have shown that volatility-scaled portfolios are often not spanned by the original portfolios, and the alphas from spanning tests are economically large. However, those alphas are not always statistically large due to the low power of univariate tests.

We further demonstrate the economic benefits of volatility-scaled portfolios through examining how the investor's opportunity set changes in a mean-variance setting. The ex post mean-variance frontier, formed with perfect knowledge of the average returns and the covariance matrix of the constituent assets, put an upper bound on the largest possible investment opportunity set for the investor. Because volatility-scaled portfolios are not spanned by the original factors, it is like by combining volatility-scaled portfolios with the original portfolios, we can expand the investor's ex post mean-variance frontier. We construct the ex post mean-variance frontiers for different sets of portfolios and show how those frontiers change when we add volatility-scaled portfolios to the mix.

Suppose we have financial assets with excess returns μ and variance-covariance matrix $\sum$. Mean-variance efficient portfolios for a target portfolio return r_0 is the solution to the following optimization problem

$$\min_{w} w^T \sum w$$

$$s.t. \quad \mu^T \sum \geq r_0, \;\; 1^T w = 1$$

Where 1^T is a conforming column of ones. The above problem was first proposed by Markowitz (1952, 1959) in his seminal papers on modern portfolio theory. To construct the mean-variance efficient frontier, we solve this problem for different values of target portfolio return r_0 to obtain portfolio weights w, then plot r_0 against the portfolio standard deviation $\sqrt{w^T \sum w}$.

Figure 8 considers the ex post mean-variance frontier formed with the Fama and French (1992) factors and momentum (Carhart, 1997). We consider 100 different portfolio target returns r_0 and solve for the mean-variance efficient portfolios. The blue curve illustrates the mean-variance frontier generated by the Fama and French (1992) factors and momentum. The green curve constructs the mean-variance frontier using a combination of the four factors and their volatility-scaled counterparts, for a total of eight portfolios. The mean-variance frontier including volatility-scaled factors appears to provide the investor with better choices. Compared to feasible portfolios on the blue frontier, the expanded green frontier allows for portfolios with lower volatility (given the same level of expected returns), higher expected returns (given the same level of volatility), or portfolios that have both lower volatility and higher returns. Portfolios between the green and blue frontiers can have higher Sharpe ratios than feasible portfolios inside the blue frontier.

[Figure 8 about here]

Figure 9 presents mean-variance frontiers formed using 63 industry portfolios (blue), or those 63 portfolios and their volatility-scaled counterparts (green). We observe a similar pattern for the two mean-variance frontiers as in Figure 8. When we combine volatility-scaled portfolios to the original industry portfolios, the mean-variance frontier moves up and to the left in the figure, expanding the investor's opportunity set. For the investor, the ability to access volatility-scaled portfolios as part of the investment mix has the potential to improve his risk-return profile. The improvement in mean-variance frontier for industry portfolios is larger compared to the factors, reflecting that lagged volatility is a better predictor of future returns for industry portfolios.

[Figure 9 about here]

4. Discussion

4.1 Possible Explanations

Leading asset pricing models often contain some form of positive risk-return tradeoff. Moreira and Muir (2017) examine four models:

1. Campbell and Cochrane (1999): habit formation
2. Wachter (2013): disaster risk
3. Bansal and Yaron (2004); Bansal, Kiku, and Yaron (2012): long-run risk
4. He and Krishnamurthy (2013): intermediary-based model

Through simulations, Moreira and Muir (2017) show that these models imply zero or negative intercepts in spanning regressions of volatility-managed portfolios onto the original portfolios. In particular, long-run risk and intermediary-based models appear to show coefficients centered around zero, whereas habit formation and disaster risk models tend to show negative intercepts. Since Moreira and Muir (2017) find positive spanning regression intercepts for the aggregate U.S. market, they assert their empirical results pose a challenge to these workhorse asset pricing models.

Our results for the Chinese stock market stand in sharp contrasts to those of Moreira and Muir (2017). We find generally negative alphas in spanning regressions of volatility-managed portfolios onto the original portfolios. Specifically, the intercept is small and negative for the aggregate A-share market, consistent with the predictions from the four leading asset pricing models.

Another possible channel to generate our empirical findings is that investors may become more risk-averse in times of high volatility. As such, those who can bear risks in these times are rewarded for having greater risk tolerance than the average investor. This channel is most plausible in a retail-dominated market, as retail investors tend to be more myopic compared to institutional investors, who may have longer investment horizons as well as greater capacity to bear short-term risk. The different institutional settings for the Chinese and U.S. stock markets may help explain the distinct portfolio management tools that are effective in each market.

4.2 Volatility or Variance?

We follow the theoretical framework of Moreira and Muir (2017) to illustrate the different interpretations associated with different portfolio management tools based on volatility.

Let r_t be the instantaneous risk-free rate. Suppose a portfolio's value R_t has conditional expected returns $r_t + \mu_t$ and volatility σ_t: $dR_t = (r_t + \mu_t)dt + \sigma_t dB_t$. Then Moreira and Muir's volatility-managed portfolio, R_t^{MM}, takes on the form $dR_t^{MM} = r_t dt + \frac{c}{\sigma_t^2}(dR_t - r_t dt)$, where *c* is the normalization constant from Equation (5). A spanning regression of excess returns of the volatility-managed portfolio R_t^{MM} onto the original portfolio R_t can be represented in continuous time as a regression of $dR_t^{MM} - r_t dt$ on $dR_t - r_t dt$. The regression coefficient is

$$\beta_{MM} = \frac{cov(dR_t^{MM} - r_t dt,\ dR_t - r_t dt)}{var(\ dR_t - r_t dt)}$$

$$= \frac{c}{E[\sigma_t^2]} \quad (9)$$

The intercept of this regression, as Moreira and Muir (2017) show, can be written as

$$\alpha_{MM} = E[dR_t^{MM} - r_t dt]/dt - \beta_{MM} E[dR_t - r_t dt]/dt$$

$$= -cov\left(\frac{\mu_t}{\sigma_t^2}, \sigma_t^2\right)\frac{c}{E[\sigma_t^2]} \quad (10)$$

Equation (10) shows the spanning regression intercept α_{MM} measures how much the price of risk and variance comove. Since the second term in (10) is positive, the covariance is typically positive for the Chinese A-share market to result in negative intercepts α_{MM}.

What is the interpretation of the regression intercept if we use volatility rather than variance to scale the portfolio? The managed portfolio would take the form $dR_t^v = r_t dt + \frac{h}{\sigma_t}(dR_t - r_t dt)$ for some constant *h*. A spanning regression of $dR_t^v - r_t dt$ on $dR_t - r_t dt$ would have the regression coefficient

$$\beta_v = \frac{cov(dR_t^v - r_t dt,\ dR_t - r_t dt)}{var(\ dR_t - r_t dt)}$$

$$= \frac{h}{E[\sigma_t]} \quad (11)$$

The spanning regression intercept can be calculated as follows

$$\alpha_v = -cov\left(\frac{\mu_t}{\sigma_t}, \sigma_t\right)\frac{h}{E[\sigma_t]} \quad (12)$$

Equation (12) shows that if we use volatility to scale portfolios rather than variance, the spanning regression intercept measures the comovement between the Sharpe ratio of the portfolio and its volatility.

What about volatility-scaled portfolios? In our proposed approach, portfolio exposure is increased when lagged volatility is high and decreased when lagged volatility is low. To construct such a portfolio, we form $dR_t^{vs} = r_t dt + \frac{\sigma_t}{k}(dR_t - r_t dt)$. A spanning regression of $dR_t^{vs} - r_t dt$ on $dR_t - r_t dt$ has the regression coefficient β_{vs}

$$\beta_{vs} = \frac{cov(dR_t^{vs} - r_t dt,\ dR_t - r_t dt)}{var(\ dR_t - r_t dt)}$$

$$= \frac{E[\sigma_t]}{k} \quad (13)$$

The intercept is then

$$\alpha_{vs} = \frac{1}{k}\ cov(\sigma_t, \mu_t) \quad (14)$$

Equation (14) shows that volatility-scaled portfolios exploit the covariance between volatility and expected returns. To the extent lagged volatility has predictive power for future expected returns, volatility-scaled portfolios are designed to exploit this effect.

The regression coefficient δ of future returns on lagged volatility also measures $cov(\sigma_t, \mu_t)$. In the spanning regressions of volatility-scaled portfolios on the original portfolios, positive α_{vs}reflect a positive covariance between lagged volatility and future returns. Portfolios with negative α_{vs} are typically the ones with a negative predictive coefficient δ.

4.Volatility Forecasts versus Lagged Volatility

Volatility-scaled portfolios are constructed by setting the portfolio exposure to be proportional to the previous month's volatility. For robustness, we consider an alternative portfolio construction using volatility forecasts rather than lagged volatility. We use a GARCH(1,1) (Engle, 1982; Bollerslev, 1986) model to produce monthly volatility forecast, then form volatility scaled portfolios f_{t+1}^{GARCH}.

$$f_{t+1}^{GARCH} = \frac{\sigma_{t+1}^{GARCH}}{\lambda} f_{t+1} \quad (15)$$

Where λ is a constant set such that f_{t+1}^{GARCH} and f_{t+1} have the same unconditional standard deviation. We then look at spanning regressions of f_{t+1}^{GARCH} on f_{t+1}. The intercepts and t-statistics are shown in Figure 10.

[Figure 10 about here]

Figure 10 is similar to Figure 7, for volatility-scaled portfolios. 53 of 69 intercepts are positive; 16 are negative. 32 intercepts are between 1% and 4%. Using GARCH volatility forecast to manage portfolio exposure expands the investor's opportunity set in much of the same way as using lagged volatility.

5. Conclusion

In this paper, we investigate the risk-return tradeoff for the Chinese A Share market. We start by trying to understand the relationship between volatility and returns. Whereas returns are negatively correlated to contemporaneous measures of volatility and change in volatility, returns are positively related to lagged volatility and change in volatility from the previous month. These results stand in contrast to the findings for the U.S. markets, where lagged volatility and current period returns do not have a clear positive relationship.

Due to the positive relationship between return and lagged volatility, Moreira and Muir's (2017) volatility-managed portfolios do not work in China. These portfolios scale back exposure when volatility is high, and scale up exposure when volatility is low. This type of portfolio management does not account for the positive return predictability from lagged volatility. As a result, volatility-managed portfolios in Chinese A Shares are spanned by the original portfolios. Spanning regressions of volatility-managed portfolios on the original portfolios mostly show negative intercept estimates. Volatility-managed portfolios do not expand the investor's opportunity set relative to the original portfolios.

Motivated by the positive relationship between lagged volatility and returns, we propose volatility-scaled portfolios, which increase portfolio exposure when volatility is high and decrease exposure when volatility is low — just the opposite of volatility-managed portfolios.

We find volatility-scaled portfolios are not spanned by the original portfolios; spanning regressions show many economically large intercepts up to 6% per year. Volatility-scaled portfolios expand the investor's opportunity set when compared to the original portfolios.

We also investigate how volatility-scaled portfolios add value for the investor in a mean-variance framework. We construct mean-variance frontiers using the original portfolios and compare to frontiers constructed when we include volatility-scaled portfolios. Mean-variance frontiers including volatility-scaled portfolios provide investors with a superior investment set that includes new feasible portfolios with higher Sharpe ratios. By combining the original portfolios with volatility-scaled portfolios, investors would achieve better risk-return tradeoffs.

Our empirical findings are consistent with the predictions of several leading asset pricing models, including habit formation of Campbell and Cochrane (1999), disaster risk of Wachter (2013), long-run risk of Bansal and Yaron (2003), and the intermediary-based model of He and Krishnamurthy (2013). We also offer an intuitive explanation through the lens of retail investors. We show how our empirical results can be interpreted through the framework in Moreira and Muir (2017), and how a volatility-scaled portfolio differs from a volatility-managed portfolio. Lastly, we show our results are unchanged if we use GARCH volatility forecasts in volatility-scaled portfolios rather than lagged volatility.

While we present novel patterns of returns and volatility, and we examine their portfolio management implications, we only provide suggestive explanations for our empirical findings. Additional empirical tests or quantitative models are needed to better understand the different performance of volatility-managed portfolios and volatility-scaled portfolios in China. These models would likely have to capture investor behavior and institutional details for the American and Chinese stock markets. An interesting research direction would be to relate market structure to the stylized facts that we document.

We explored time-series properties of risk and return, comparing volatility and returns for single return series. While we present evidence of a positive relationship between risk and return in this setting, it is not clear what the cross-sectional implications are for our findings. Is volatility a priced risk factor in the cross section of stock returns? Can individual stock volatility forecast that stock's future returns? These are questions we leave to future research.

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Appendix

GICS code and corresponding industries (effective September 1, 2016)

Industry		Industry	
101010	Energy Equipment & Services	302010	Beverages
101020	Oil, Gas & Consumable Fuels	302020	Food Products
151010	Chemicals	302030	Tobacco
151020	Construction Materials	303010	Household Products
151030	Containers & Packaging	303020	Personal Products
151040	Metals & Mining	351010	Health Care Equipment & Supplies
151050	Paper & Forest Products	351020	Health Care Providers & Services
201010	Aerospace & Defense	351030	Health Care Technology
201020	Building Products	352010	Biotechnology
201030	Construction & Engineering	352020	Pharmaceuticals
201040	Electrical Equipment	352030	Life Sciences Tools & Services
201050	Industrial Conglomerates	401010	Banks
201060	Machinery	401020	Thrifts & Mortgage Finance
201070	Trading Companies & Distributors	402010	Diversified Financial Services
202010	Commercial Services & Supplies	402020	Consumer Finance
202020	Professional Services	402030	Capital Markets
203010	Air Freight & Logistics	402040	Mortgage Real Estate Investment; Trusts (REITs)
203020	Airlines	403010	Insurance
203030	Marine	451010	Internet Software & Services
203040	Road & Rail	451020	IT Services
203050	Transportation Infrastructure	451030	Software
251010	Auto Components	452010	Communications Equipment
251020	Automobiles	452020	Technology Hardware, Storage & Peripherals
252010	Household Durables	452030	Electronic Equipment, Instruments & Components
252020	Leisure Products	453010	Semiconductors & Semiconductor Equipment
252030	Textiles, Apparel & Luxury Goods	501010	Diversified Telecommunication Services
253010	Hotels, Restaurants & Leisure	501020	Wireless Telecommunication Services
253020	Diversified Consumer Services	551010	Electric Utilities
254010	Media	551020	Gas Utilities
255010	Distributors	551030	Multi-Utilities
255020	Internet & Direct Marketing Retail	551040	Water Utilities
255030	Multiline Retail	551050	Independent Power and Renewable Electricity
255040	Specialty Retail	601010	Equity Real Estate Investment Trusts; (REITs)
301010	Food & Staples Retailing	601020	Real Estate Management & Development

Table 1: **Summary Statistics of Benchmark Factor Returns, 1998-2015**. The market risk premium, Rm-Rf, is the value-weight one-month return on publicly listed A-share stocks on the Shanghai and Shenzhen exchanges. Weights are monthly market capitalizations. Rf is the three-month Chinese household deposit rate. We form size (SMB) and value (HML) factors using the Fama and French (1992) methodology, rebalancing the portfolios annually or monthly. The momentum (MOM) factor is rebalanced monthly. T-statistics are reported in parentheses.

	Rm-Rf	SMB	HML	MOM
Annual Sort	0.82%	0.86%	0.41%	
	(1.4)	(2.8)	(1.6)	
Monthly Sort		1.15%	1.05%	-0.22%
		(3.4	(3.9)	(-0.8)

Table 2: **Contemporaneous Relationship between Volatility and Returns**. The top panel shows results for regressions of long-short factor portfolio returns on the contemporaneous month volatility, $f_t = a + b\sigma_t + \eta_t$. The bottom panel shows results for regressions of portfolio returns onto the contemporaneous change in volatility, $f_t = a^{ch} + b^{ch}\Delta\sigma_t + \eta_t^{ch}$. T-statistics are shown in parentheses. The right column shows pooled regression coefficients, including portfolio fixed effects. Standard errors for the pooled regressions are clustered by portfolio and time.

	RmRf	SMB_Annual	HML_Annual	MOM	SMB_Monthly	HML_Monthly	Pooled
	Returns on Volatility						
b	-0.06	-0.11	0.07	-0.14	-0.05	0.19	-0.03
	(-1.4)	(-1.7)	(1.4)	(-2.6)	(-0.8)	(4.1)	(-0.8)
R^2	0.8%	1.2%	0.9%	2.8%	0.3%	6.6%	0.8%
	Returns on Change in Volatility						
b^{ch}	-0.11	-0.38	0.15	0.10	-0.40	0.12	-0.10
	(-2.1)	(-5.7)	(2.4)	(1.6)	(-5.8)	(2.2)	(-2.3)
R^2	1.8%	12.4%	2.4%	1.1%	12.3%	2.0%	1.9%

Table 3: **Relationship between Lagged Volatility and Future Returns**. The top panel shows results for regressions of factor portfolios on the previous month volatility, $f_t = \gamma + \delta\sigma_{t-1} + \varepsilon_t$. The bottom panel shows results for regressions of portfolio returns on the change in lagged volatility, $f_t = \gamma^{ch} + \delta^{ch}\Delta\sigma_{t-1} + \varepsilon_t^{ch}$. T-statistics are shown in parentheses. The right column shows pooled regression coefficients, including portfolio fixed effects. Standard errors for the pooled regressions are clustered by portfolio and by time.

	RmRf	SMB_Annual	HML_Annual	MOM	SMB_Monthly	HML_Monthly	Pooled
	Returns on Lagged Volatility						
δ	0.01	0.21	-0.02	-0.22	0.29	0.10	0.04
	(0.3)	(3.3)	(-0.4)	(-4.1)	(4.4)	(2.1)	(1.2)
R^2	0.0%	4.6%	0.1%	6.9%	7.6%	1.8%	0.9%
	Returns on Change in Lagged Volatility						
δ^{ch}	0.02	0.04	0.01	-0.05	0.09	0.08	0.03
	(0.4)	(0.6)	(0.2)	(-0.8)	(1.2)	(1.5)	(0.8)
R^2	0.1%	0.1%	0.0%	0.3%	0.6%	1.0%	0.8%

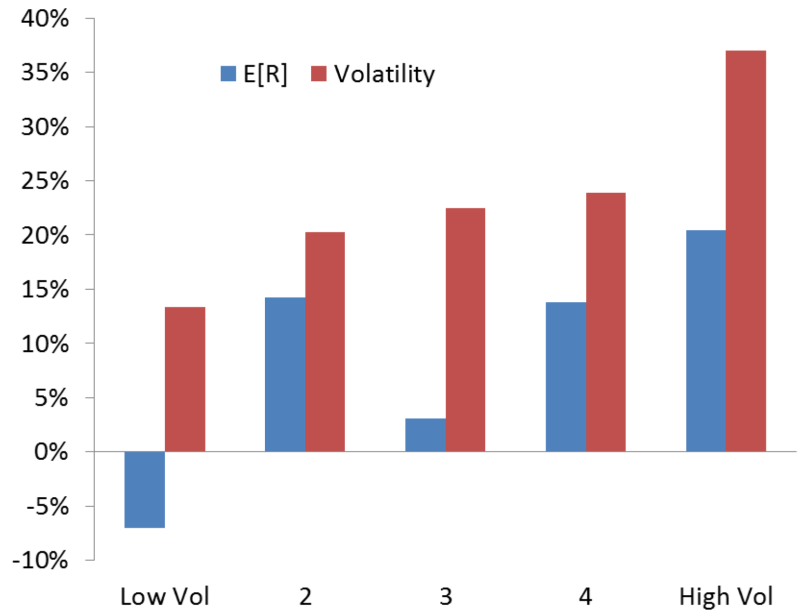

Figure 1: **Volatility Quintiles, Value-Weight Chinese A Shares.**

We sort monthly-realized volatility of the returns on the value-weight A Shares stock market into five buckets and track the portfolio behavior for the following month. Expected returns and volatility are annualized.

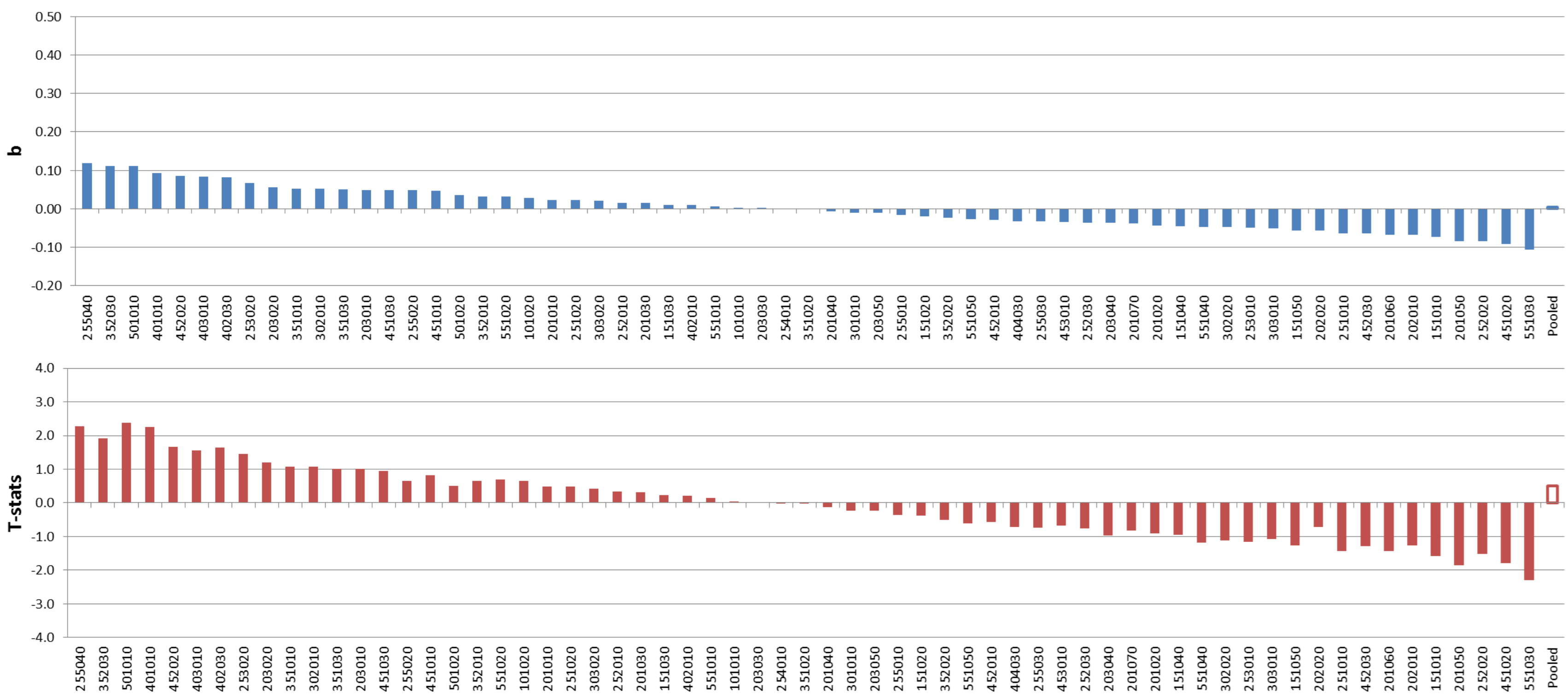

Figure 2: **Regression of Monthly Returns on Volatility**.

We regress industry portfolio returns on the contemporaneous month volatility, $f_t = a + b\sigma_t + \eta_t$. Regression coefficients b and t-statistics are sorted from the largest to smallest b. The rightmost (unshaded) bar shows the pooled regression coefficient including industry fixed effects. T-statistics clustered by portfolio and by time.

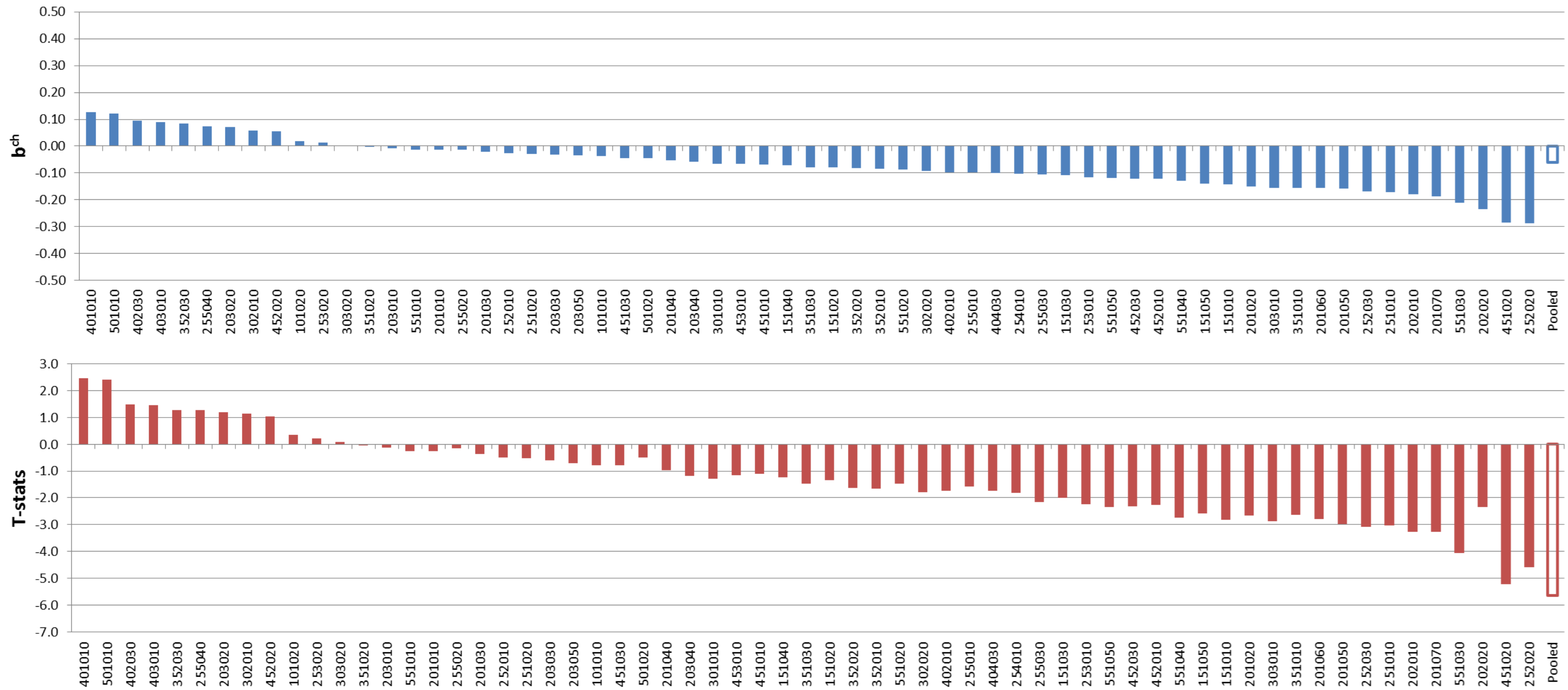

Figure 3: **Regression of Monthly Returns on Change in Volatility**.

We regress industry portfolio returns on the change in volatility. $f_t = a^{ch} + b^{ch}\Delta\sigma_t + \eta_t^{ch}$. Regression coefficients b^{ch} and t-statistics are sorted from the largest to smallest b^{ch}. The rightmost (unshaded) bar shows the pooled regression coefficient including industry fixed effects. T-statistics clustered by portfolio and by time.

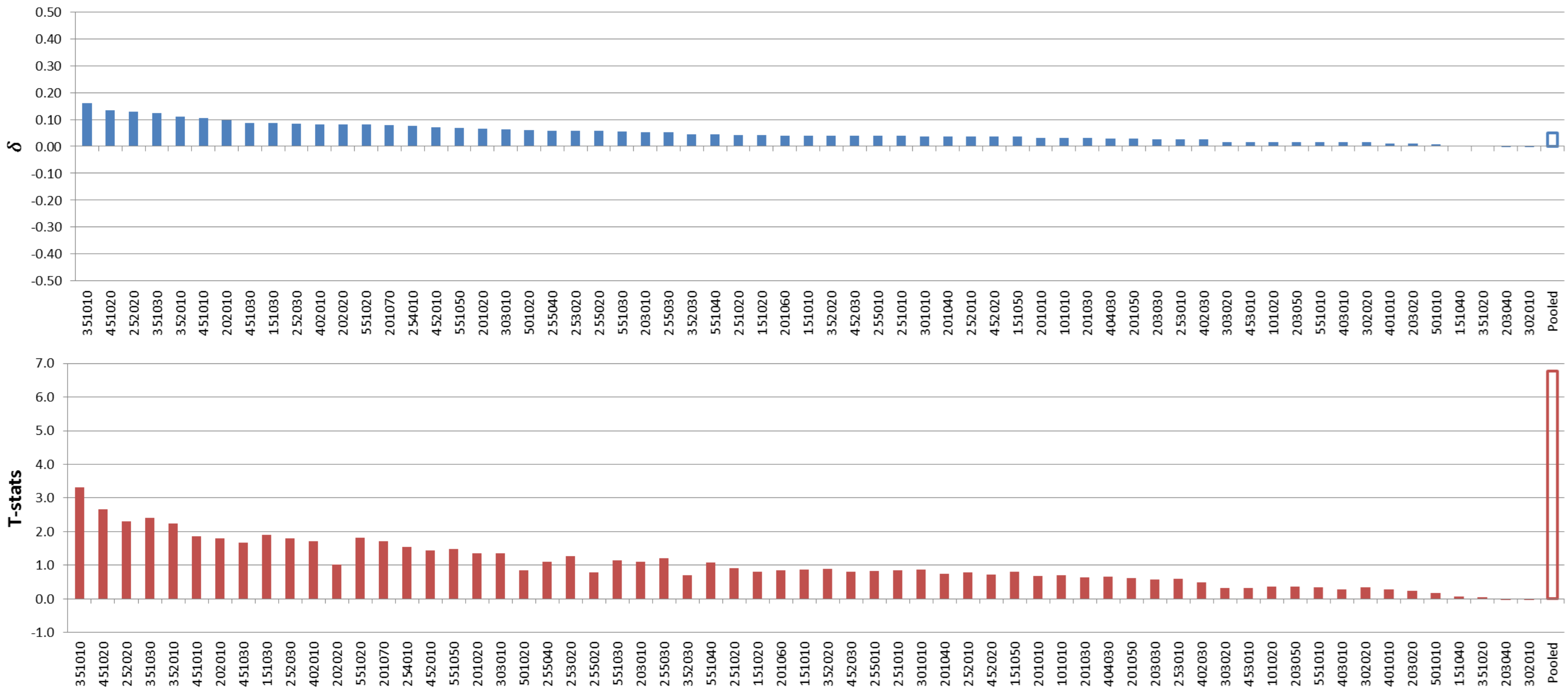

Figure 4: **Regression of Monthly Returns on Lagged Volatility**.

We regress industry portfolio returns on portfolio volatility from the previous month. $f_t = \gamma + \delta\sigma_{t-1} + \varepsilon_t$. Forecasting coefficients δ and t-statistics are sorted from the largest to smallest δ. The rightmost (unshaded) bar shows the pooled regression coefficient including industry fixed effects. T-statistics clustered by portfolio and by time.

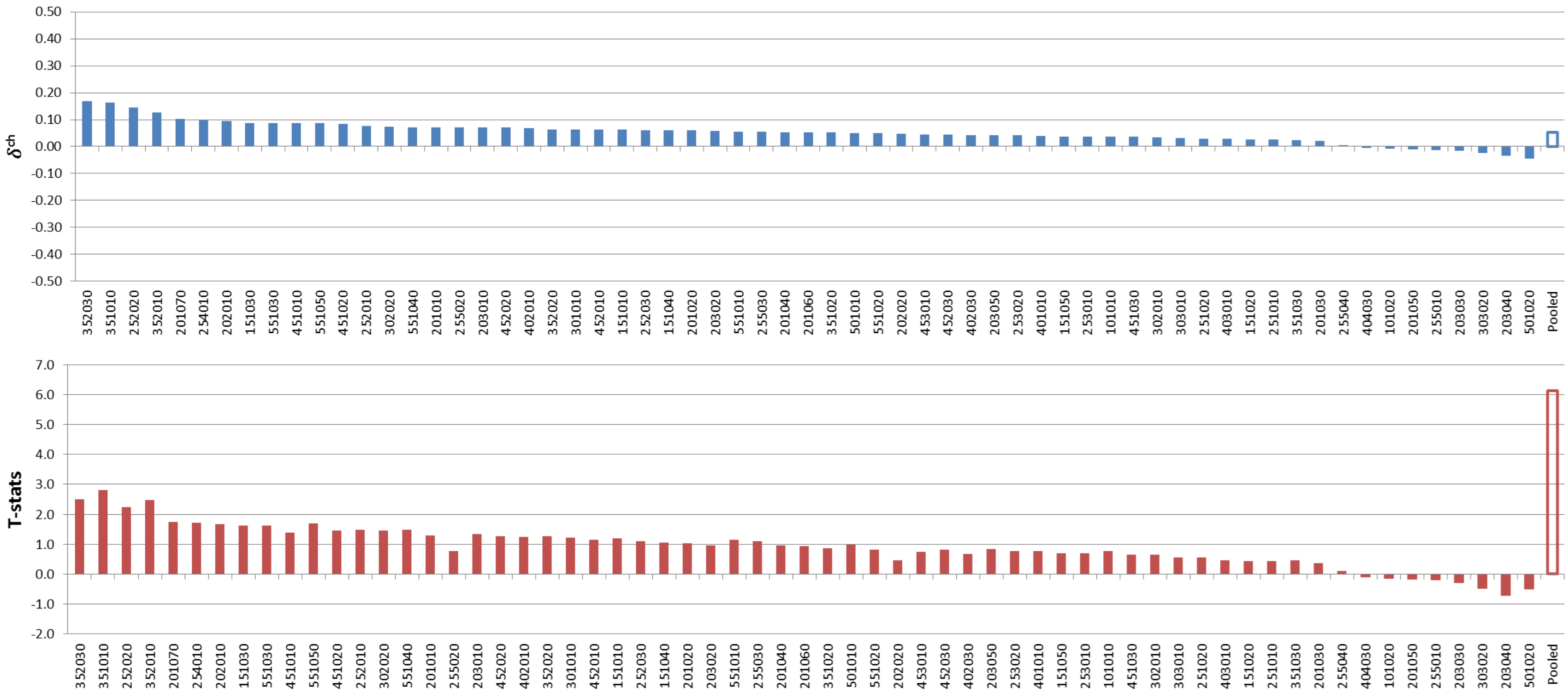

Figure 5: **Regression of Monthly Returns on Lagged Volatility Changes**.

We regress industry portfolio returns on the first difference in portfolio volatility from the previous month. $f_t = \gamma^{ch} + \delta^{ch} \Delta\sigma_{t-1} + \varepsilon_t^{ch}$. Forecasting coefficients δ^{ch} and t-statistics are ordered from the largest to smallestδ^{ch}. The rightmost (unshaded) bar shows the pooled regression coefficient including industry fixed effects. T-statistics clustered by portfolio and by time.

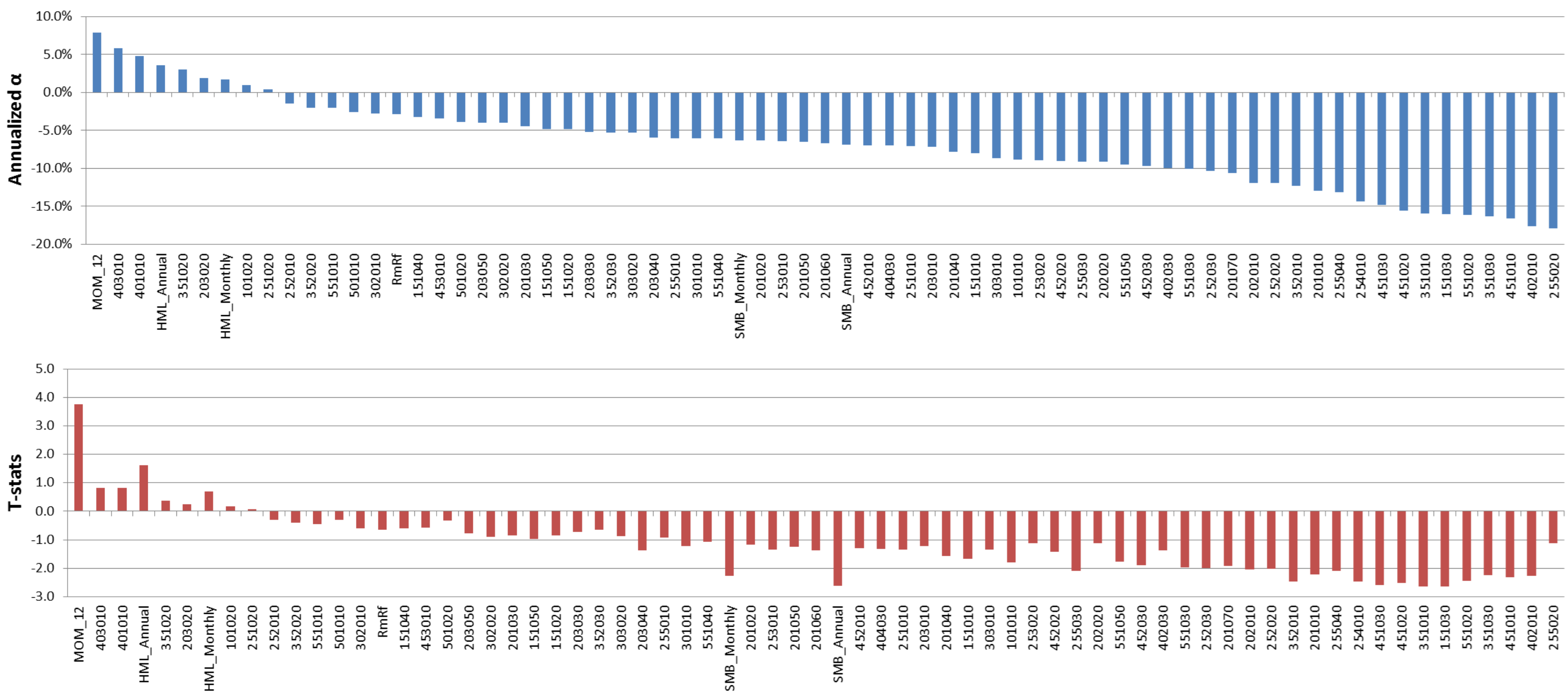

Figure 6: **Spanning Regressions of Volatility-Managed Portfolios on the Original Portfolios**.

We construct volatility-managed portfolios of Moreira and Muir (2017): $f_{t+1}^{MM} = \frac{c}{\sigma_t^2} f_{t+1}$, where f_{t+1}^{MM} is the Moreira and Muir (2017) volatility-managed portfolio. f_{t+1} is the original portfolio. σ_t^2 is the variance estimated using last month's daily observations. c is a constant set such that f_{t+1}^{MM} and f_{t+1} have the same unconditional standard deviation. We then regress volatility-managed portfolio f_{t+1}^{MM} on the original portfolio f_{t+1}: $f_{t+1}^{MM} = \alpha + \beta f_{t+1} + \varepsilon_{t+1}^{MM}$. Annualized figures of α and t-statistics are sorted by α.

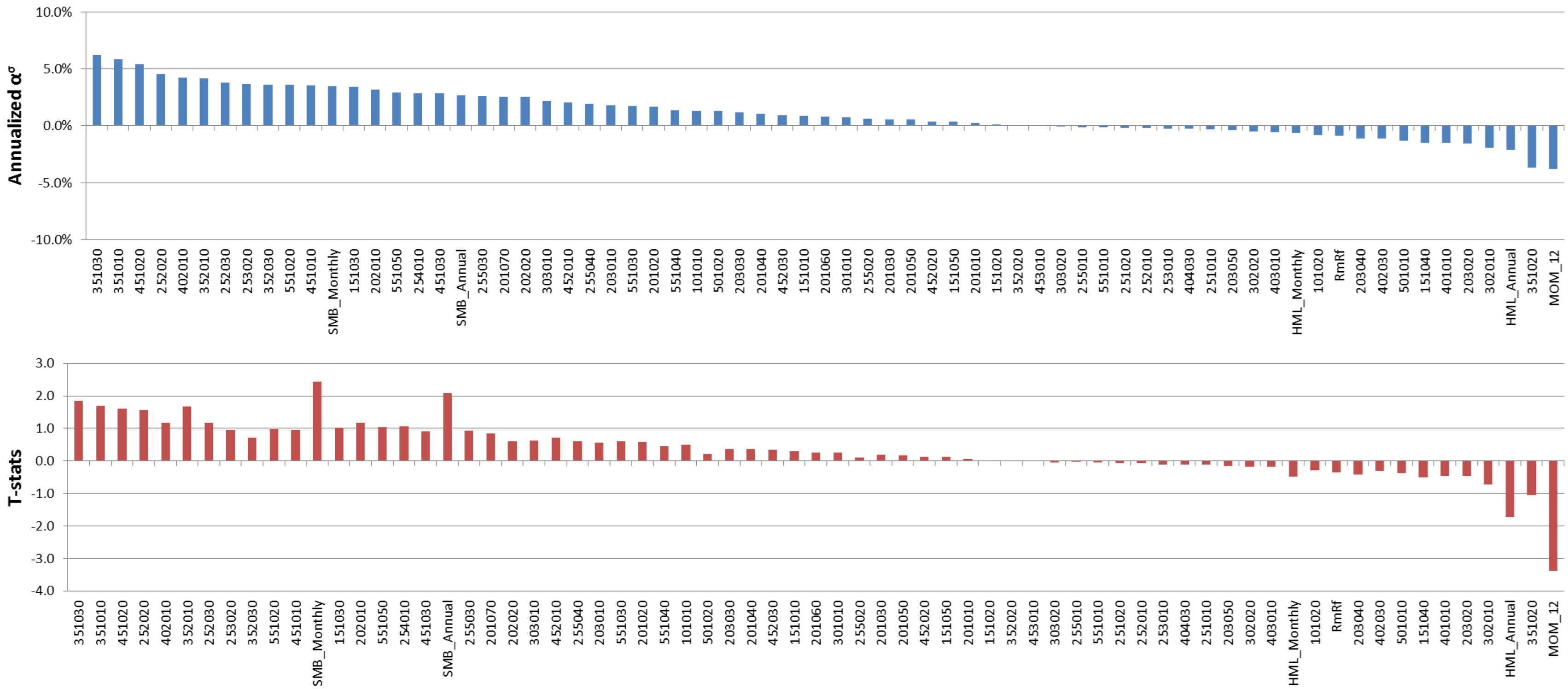

Figure 7: **Spanning Regressions of Volatility-Scaled Portfolios on the Original Portfolios**.

We construct volatility-scaled portfolios as follows: $f_{t+1}^{\sigma} = \frac{\sigma_t}{k} f_{t+1}$, where f_{t+1}^{σ} is the volatility-scaled portfolio. f_{t+1} is the original portfolio. σ_t is the standard deviation estimated using daily observations of f_t. k is a constant such that f_{t+1}^{σ} and f_{t+1} have the same unconditional standard deviation. We then regress volatility-scaled portfolio f_{t+1}^{σ} on the original portfolio f_{t+1}: $f_{t+1}^{\sigma} = \alpha^{\sigma} + \beta^{\sigma} f_{t+1} + \varepsilon_{t+1}^{\sigma}$. Annualized figures of α^{σ} and t-statistics are sorted by α^{σ}.

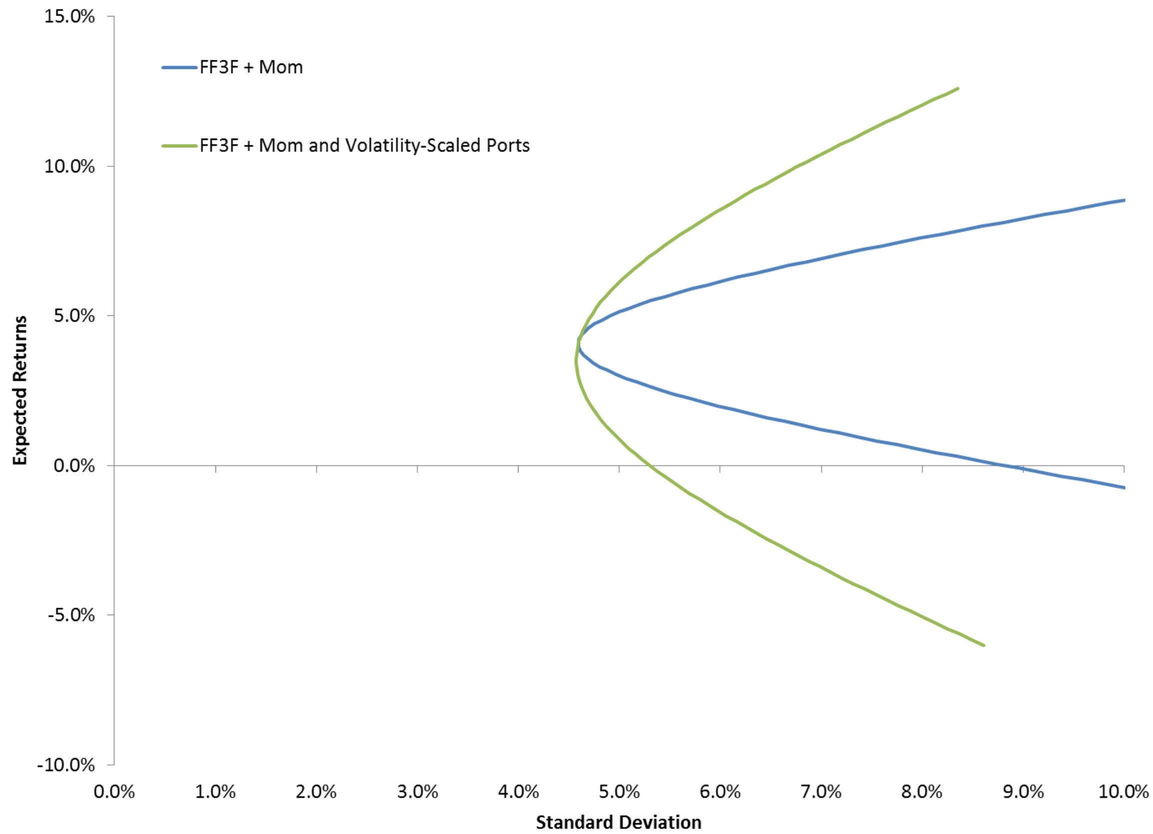

Figure 8: **Ex Post Mean-Variance Frontiers for Factors**.

We construct the ex post mean-variance frontier for two sets of long-short factor portfolios. The blue curve is the mean-variance frontier constructed from market, size, value, and momentum factors (Fama and French, 1992; Carhart, 1997). The green curve is constructed from the same factors as the blue curve, plus the volatility-scaled versions of the four factors.

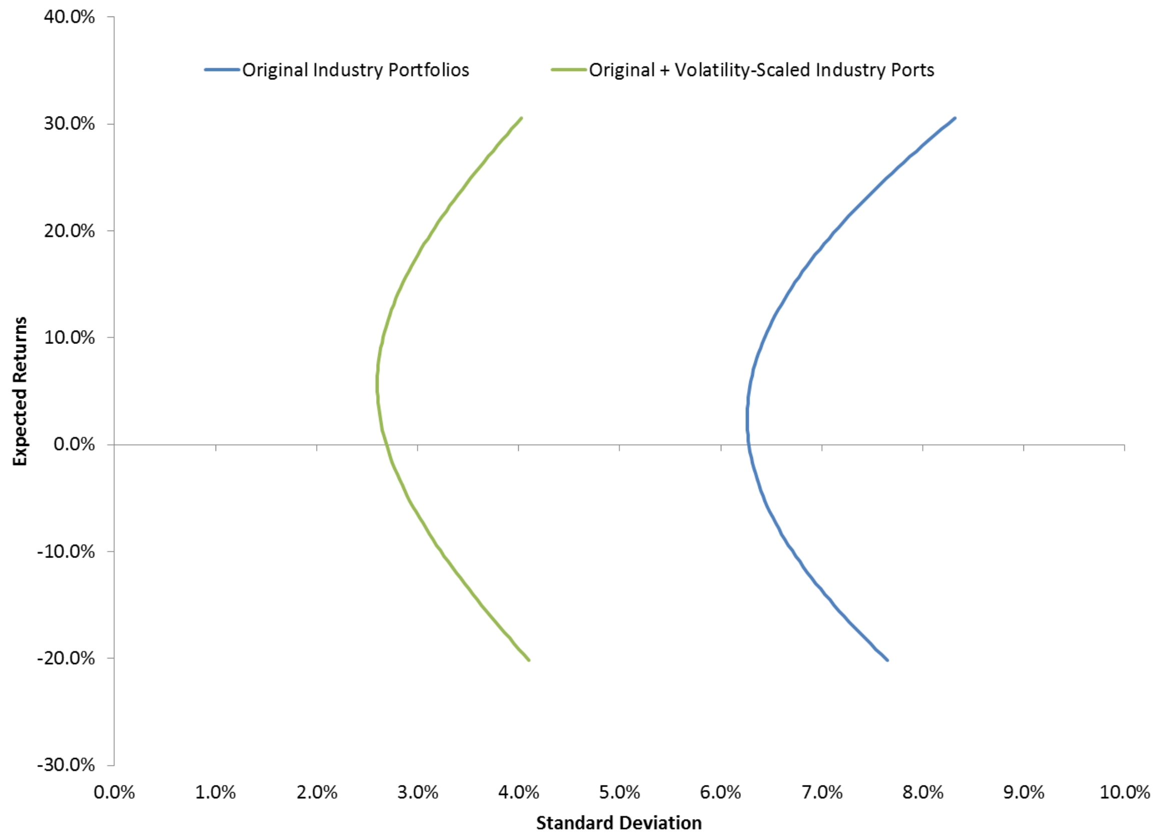

Figure 9: **Ex Post Mean-Variance Frontiers for Industry Portfolios**.

We construct the ex post mean-variance frontier for two sets of industry portfolios. The blue curve is the mean-variance frontier constructed from 63 industry portfolios. The green curve is constructed from the original 63 industry portfolios plus their volatility-scaled versions.

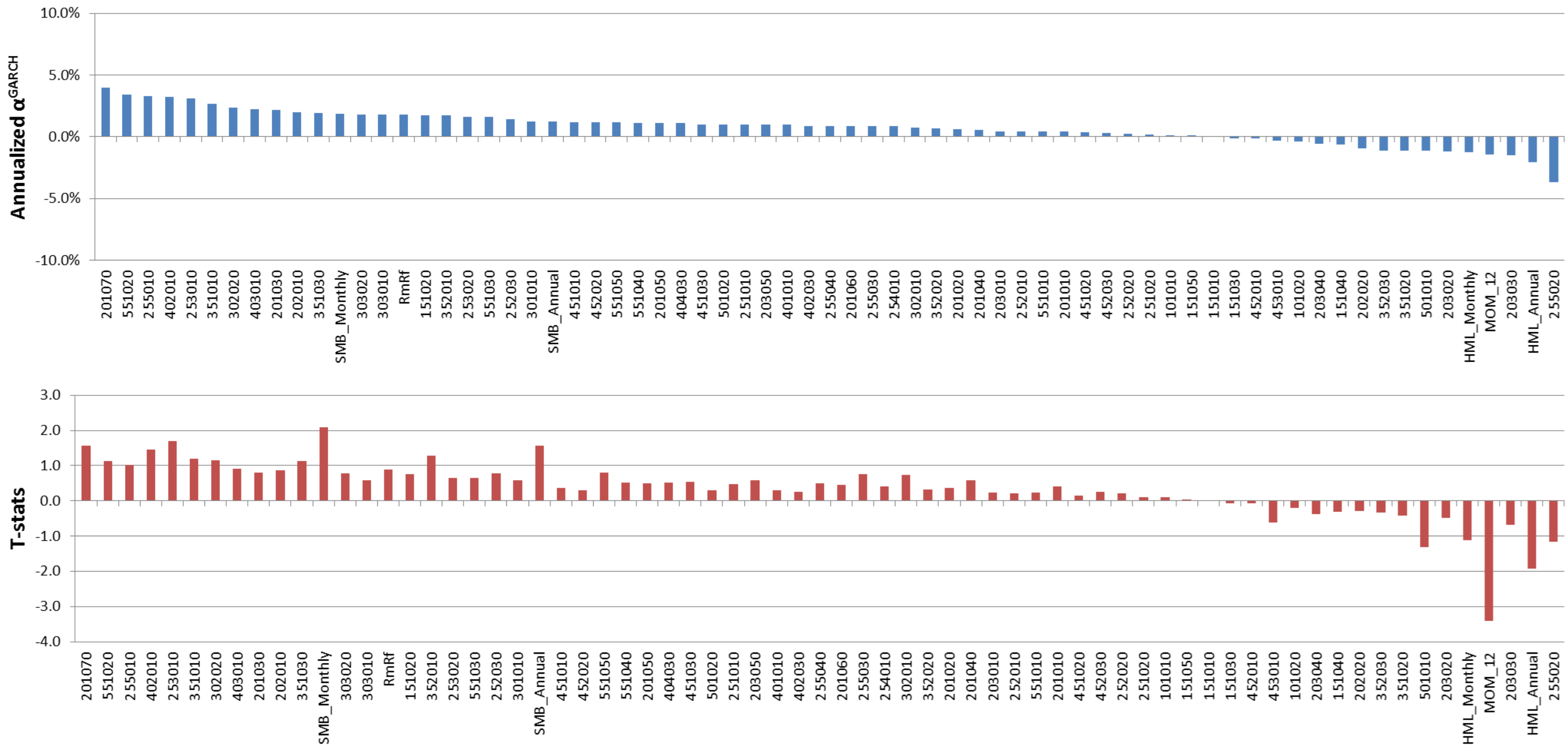

Figure 10: **Spanning Regressions of Volatility-Scaled Portfolios using GARCH Forecasts**.

We construct volatility-scaled portfolios as follows: $f_{t+1}^{GARCH} = \frac{\sigma_{t+1}^{GARCH}}{\lambda} f_{t+1}$, where f_{t+1}^{GARCH} is the volatility-scaled portfolio using GARCH forecasts. f_{t+1} is the original portfolio. σ_{t+1}^{GARCH} is the one-step ahead forecast of monthly volatility using a GARCH(1,1). λ is a constant such that f_{t+1}^{GARCH} and f_{t+1} have the same unconditional standard deviation. We then regress volatility-scaled portfolio f_{t+1}^{GARCH} on the original portfolio f_{t+1}: $f_{t+1}^{GARCH} = \alpha^{GARCH} + \beta^{GARCH} f_{t+1} + \varepsilon_{t+1}^{GARCH}$. Annualized figures of α^{GARCH} and t-statistics are sorted by α^{GARCH}.