A Brain-Focused Approach to the Equity Premium Puzzle

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Abstract

We model the human brain as the ultimate scarce, efficient, and rational resource that first must optimize on itself before optimizing on the resources available in the external world. We show that a new unified explanation for the equity premium puzzle, countercyclical equity premia, value effect, and size effect emerges in the enriched framework.

JEL Classification G12, G10

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The fact that human brain is a limited resource is generally acknowledged; however, the scarcity of brain resources is seldom explicitly taken into account in economics and finance literature.[1] In this article, based on recent findings from brain sciences, we push the notion of scarcity inside the human brain and model the brain as solving two optimization problems instead of one, which are: (i) optimal resource allocation in the brain and (ii) expected utility maximization. We show that a new explanation for the equity premium puzzle emerges in the enriched framework, which also generates countercyclical equity premia as well as value and size effects. Hence, several anomalies in the standard framework are potentially reconciled in the enriched framework.

The equity premium puzzle (Mehra and Prescott 1985, Hansen and Jagannathan 1991) has sparked a large literature in macro-finance that explores a wide range of alternative preferences and market structures in an attempt to explain the puzzle.[2] In this article, we introduce a new approach to addressing the equity premium puzzle. This novel approach incorporates optimal resource allocation inside the human brain. One of the most intriguing findings from decision neuroscience is that the brain separately encodes reward and risk while evaluating a gamble (see the discussion in Bossaerts (2009))[3]. In addition, research in brain sciences has established that when the brain is engaged in multiple tasks then each task is assigned to a separate system of neurons with each system competing for scarce brain resources that are allocated by the 'central executive system' (CES) located in the lateral prefrontal cortex (see Alonso et al (2014) and references therein). It follows that

[1] McKenzie (2018) argues that a brain-focused approach could potentially lead to an integration of neoclassical and behavioral economics. Siddiqi and Murphy (2020) show that adjusting CAPM for optimal resource allocation in the brain provides a unified explanation for anomalies such as size, value, and momentum.

[2] A sample of this literature includes habits (Campbell and Cochrane 1999a, 1999b), recursive utility (Epstein and Zin 1989), long-run risks (Bansal and Yaron 2004, Bansal, Kiku, and Yaron 2012), idiosyncratic risk (Constantinides and Duffie 1996), heterogeneous preferences (Garleanu and Panageas 2015), rare disasters (Reitz 1989, Barro 2006), non-separable utility across goods (Piazzesi, Schneider, and Tuzel 2007), institutional finance (Brunnermeir 2009, Krishnamurthy and He 2013), ambiguity aversion (Hansen and Sargent 2001), and behavioral finance (Shiller 1981, 2014) among others.

[3] Expected reward is encoded in the subcortical projection areas of the dopamine neurons, in particular the ventral striatum, whereas brain regions involved in risk (variance) encoding include right and left insula, and thalamus

expected reward and risk are considered separate tasks in the brain with their own dedicated systems of neurons competing for scarce resources.

Opening the black-box of brain processes yields further insights regarding information processing in a resource-rational brain. A key finding from neuroscience is that, when information reaches the brain, a brain template or schema is first activated, which influences information absorption.[4] The schema provides starting points for relevant forecasts. Brain resources are then optimally allocated to adjusting these starting points (Lieder et al 2018, Lieder and Griffiths 2020) with resource allocation dependent on relative task importance and task complexity (Alonso et al 2014). In this framework, rational expectations correspond to a special case, which is achieved if sufficient brain resources are available.

In this article, we show that allowing for the possibility that sufficient brain resources may not be available changes the upper-bound on the Sharpe-ratio. This change is sufficiently large to explain the equity premium puzzle. The traditional bound is recovered when the resource constraint in the brain does not bind. The generalized upper-bound is countercyclical suggesting a potential explanation for the observed countercyclical equity premia as well (Cochrane 2017, Fama and French 1989). In addition, the cross-sectional variations in Sharpe-ratios are consistent with value and size effects.

This article is organized as follows. In section 1, we discuss neuroscience and cognitive science research on schemas and the role that they play in information processing. Section 2 adjusts asset pricing for reliance on schemas and shows how a unified explanation for high equity premia that are countercyclical as well as size and value effect emerges. Section 3 concludes with directions for future research.

1. Resource-Rational Brain

Research is decision-neuroscience has shown that, when information reaches the human brain, a brain-template or schema is first activated, which influences information

[4] See section 1 for a discussion of neuroscience evidence on schemas and their role in information processing in a resource-rational brain.

absorption.[5] Schema is a cluster of related pieces of information stored in the brain. It provides a mental framework that facilitates processing of new information.[6] The brain creates a schema by integrating similar experiences in an abstract representation or a generalization, which provides useful starting points for expectations. For example, when you see a dog in your neighbour's front yard, then a schema of a pet (formed by prior experiences) may be activated. Such a schema provides useful starting points regarding what to expect in this situation. Brain resources are then spent in adjusting these starting points to the dog that you just encountered. For example, if the dog looks aggressive then the schema generated expectation of a friendly interaction may need to be amended. Schemas, because they provide useful starting points for expectations in a particular situation, are great economizing tools critically important for the resource-rational brain.[7] It has been suggested in the literature that Autism is a condition associated with the break-down in such schema creation, which overloads the brain, adversely affecting decision making. [8]

We posit that the brain clusters similar firms together and creates a schema for the cluster based on experience with the firms in the cluster. Such experiences are integrated into an abstract representation or a generalization, which provides useful starting points for expectations. Limited brain resources are then optimally allocated to adjustment tasks based on relative task importance and task complexity.

Neuroscience evidence establishes that the human brain separately encodes expected reward and risk (Bossaerts 2009, Fukunaga et al 2018 among others). Expected reward is encoded in the subcortical projection areas of the dopamine neurons, in particular the ventral striatum, whereas brain regions involved in risk (variance) encoding include right

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[5] A sample of large and growing literature which explores the neural basis of schemas and their role in information absorption includes Tse et al (2007), van Kesteren et al (2010), Tse et al (2011), van Kesteren et al (2012), Ghosh and Gilboa (2014), Ghosh et al (2014), Brod et al (2015), Spalding et al (2015), Sweegers et al (2015), Gilboa and Marlatte(2017), and Ohki and Takei (2018).

[6] See Hampson and Morris (1996), Anderson (2000), and Pankin (2013) for an overview of schema theory in cognitive science literature. The concept of schemas goes back to the early history of cognitive science as a discipline of inquiry (Bartlett 1932, Bransford and Johnson 1972, Anderson and Pearson 1984).

[7] Brian imaging studies show that schemas lead to rapid assimilation of consistent information (see Tse et al 2007, Gilboa and Marlatte 2017. See Ohki and Takei (2018) and references therein)

[8] See Patry and Horn (2019) for a review of this literature.

and left insula, and thalamus.[9] The executive part of the brain then constructs value from the statistics of gambles (Bossaerts 2009).[10] As discussed in Alonso et al (2014), research in brain sciences has established that when the brain is engaged in multiple tasks, then each task is assigned to a separate system of neurons. Such systems compete for scarce brain resources that are allocated by the 'central executive system' (CES) located in the lateral prefrontal cortex of the brain. As reward and risk are encoded separately in the brain (Bossaerts 2009), it follows then these tasks are assigned to different systems of neurons that compete for scarce brain resources. A binding resource constraint implies that sufficient resources may not get allocated to one or both tasks.

In the next section, we adjust the standard asset pricing model for optimal resource allocation of scarce resources in the brain.

2. Asset Pricing

We take the standard consumption-based asset pricing approach and add a twist to it: *schema-creation*. As standard, we assume that investor behavior is accounted for by a representative investor who maximizes utility over current and future consumption:

$$U(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})] \tag{2.1}$$

where c_t is consumption at t.

Using w_t to denote investor wealth at t, p_{it} to denote price of stock i at t, n_i for the number of shares of stock i in the portfolio, and x_{it+1} to denote the payoff from i at $t+1$:

$$c_t = w_t - \sum_i n_i p_{it}$$

$$c_{t+1} = w_{t+1} + \sum_i n_i x_{it+1}$$

[9] There is some evidence that uncertainty in payoff distributions is processed by the brain in the same way as known risks through probability assessments/assignments (Nagel et al. 2018).

[10] The ventromedial prefrontal cortex is the area of the brain involved in carrying out an integrated valuation analysis of risk and return.

The above utility maximization results in the following key asset pricing equation:

$$p_{it} = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{it+1}\right] = E_t[M_{t+1}x_{it+1}] \tag{2.2}$$

where $M_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the stochastic discount factor/marginal rate of substitution. If the gross risk-free return is R_F, then it follows that: $E_t[M_{t+1}] = \frac{1}{R_F}$

(2.2) can be expanded as:

$$p_{it} = \frac{E_t[x_{it+1}]}{R_F} + Cov_t(M_{t+1}, x_{it+1}) \tag{2.3}$$

2.1 Schema Creation

When information about a firm arrives, a relevant schema is activated. The schema provides starting points for expectations. Brain resources are then optimally allocated to the adjustment tasks by the CES based on relative task importance and task complexity. We follow Alonso et al (2014) and Siddiqi and Murphy (2020) in specifying the adjustment mechanism below.

We assume that the cashflow analysis has two tasks. The first task is estimating expected cashflows or earnings of the firm. We refer to this task as Task 1. The second task is estimating risk, which we denote by Task 2. For simplicity, all other tasks that the brain is performing at the time of analysis are aggregated together. We refer to this aggregate as Task 3. We assume that each task is assigned to a separate brain system (of neurons), which alone is responsible for the task. These systems make resource demands to the CES with task performance dependent on actual resources allocated to the system. That is, resource deficit implies underperformance in the task.

A schema is created in the brain by integrating related experiences together into an abstract representation or a generalization. For a cluster of similar firms, $l = 1,2,3, \ldots., L$, a schema, S, is created by integrating experiences with the firms in the cluster. A person may have more experience with some firms and less experience with other firms in the cluster; hence, the contributions from firms in the cluster to the schema are unlikely to be equal. To account for such differences, we attention-weight or importance-weight contributions from various firms in the cluster. We denote these weights by v_l.

We posit that the schema relevant for cashflow analysis is given by the following set:

$S = \{earnings, risk\}$

That is, the schema, S, provides starting points for expectations regarding the following two attributes: earnings and risk.

We use π_{t+1}^{i} to denote the cashflow or earnings level of firm i at $t+1$. We denote the earnings expectations generated by the schema S based on the information set I by $E(\pi_{t+1}|S, I)$. Experience with a particular firm, l, shapes the schema based on the importance-weight, v_l, assigned to it by the brain. We take the simplest aggregation approach and set:

$$E(\pi_{t+1}|S, I) = \sum_{l \in S} v_l E\left(\pi_{t+1}^{l} | \, I\right) \tag{2.4}$$

where v_l are the importance-weights that satisfy $\sum_l v_l = 1$. Aggregation is done over the firms in the cluster.

Similarly, we define the risk expectations generated by the schema S based on the information set I by $Cov\big((\pi_{t+1}, M_{t+1}))|S, I\big)$ where $M_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the stochastic discount factor/marginal rate of substitution.

$$Cov\big((\pi_{t+1}, M_{t+1})|S, I\big) = \sum_{l \in S} v_l Cov\left(\left(\pi_{t+1}^{l}, M_{t+1}\right)|I\right) \tag{2.5}$$

Hence, when information I arrives, and schema S is activated, the starting points for earnings and risk expectations are given in the following set:

$$S = \{earnings, risk\} = \left\{\sum_{l \in S} v_l E\left(\pi_{t+1}^{l} | \, I\right), \sum_{l \in S} v_l Cov\left(\left(\pi_{t+1}^{l}, M_{t+1}\right)|I\right)\right\} \tag{2.6}$$

Task 1 involves using the schema-generated forecast of earnings as a starting point and spending brain resources in an attempt to adjust it appropriately:

$$E'\left(\pi_{t+1}^{i}|S, I\right) = \sum_{l \in S} v_l E\left(\pi_{t+1}^{l} | \, I\right) - m_1 D_1 \tag{2.7}$$

where $D_1 = \sum_{l \in S} v_l E\left(\pi_{t+1}^l | I\right) - E\left(\pi_{t+1}^i | I\right)$ is the correct adjustment needed, and m_1 is the fraction of correct adjustment achieved.

Similarly, Task 2 is:

$$Cov'\left(\left(\pi_{t+1}^i, M_{t+1}\right)|S, I\right) = \sum_{l \in S} v_l Cov\left(\left(\pi_{t+1}^l, M_{t+1}\right)|I\right) - m_2 D_2 \quad (2.8)$$

where $D_2 = \sum_{l \in S} v_l Cov\left(\left(\pi_{t+1}^l, M_{t+1}\right)|I\right) - Cov\left(\left(\pi_{t+1}^i, M_{t+1}\right)|I\right)$ is the correct adjustment needed, and m_2 is the fraction of correct adjustment achieved.

We follow Alonso et al (2014) and Siddiqi and Murphy (2020) in assuming that each system only cares about its own performance and demands resources from the CES. The resources that can be allocated to each system, $s \in \{1, 2, 3\}$, are in the set $\emptyset_s = [0, \overline{\varphi_s}]$. The amount of resources for perfect task completion is $\varphi_s \in \emptyset_s$. The amount of resources the CES allocates to a system is y_s. A system demands from the CES that it is allocated $y_s = \varphi_s$. We assume that there is a benefit function $\vartheta_s(y_s; \varphi_s)$ associated with each task that the CES computes. The benefit function takes its maximum value when $y_s = \varphi_s$. When $y_s < \varphi_s$, there is a loss. When there are too many resources, $y_s > \varphi_s$, there is no benefit. It could even be damaging as too much attention could be counterproductive. In any case, we assume that the benefit function is non-increasing when $y_s \geq \varphi_s$.

As in Alonso et al (2014) and Siddiqi and Murphy (2020), we define the following benefit function (without loss of generality):

$$\vartheta_s(y_s; \varphi_s) = \begin{cases} \alpha_s u_s(y_s - \varphi_s) & if\ y_s \leq \varphi_s \\ 0 & if\ y_s > \varphi_s \end{cases} \quad (2.9)$$

where $u_s(0) = 0$, $u_s'(0) = 0$, $u_s'(z) > 0$, and $u_s''(z) < 0$ for all $z < 0$.

(2.9) formalizes the idea that smaller the gap between resources needed and resources allocated, greater the benefit from the task.

We define m_1 and m_2 as follows:

$$m_1 = \frac{y_1}{\varphi_1} \quad (2.10)$$

$$m_2 = \frac{y_2}{\varphi_2} \tag{2.11}$$

So, m_1 and m_2 are fractions of required resources allocated to Task 1 (earnings forecast) and Task 2 (risk forecast) respectively, which is taken to be the same as the fraction of correct adjustment without loss of generality. When sufficient resources are made available to a task by the CES, the adjustment task is perfectly completed leading to rational expectations. However, when there is a resource shortfall, the adjustment process is affected in proportion with the deficit.

To set-up the optimization problem involving brain resources, we take the same approach as taken in Alonso et al (2014) and Siddiqi and Murphy (2020) and assume that the CES in the brain solves the following:

$$\max_{\{y_1, y_2, y_3\}} \vartheta_1(y_1; \varphi_1) + \vartheta_2(y_2; \varphi_2) + \vartheta_3(y_3; \varphi_3)$$

$$s.t \quad y_1 + y_2 + y_3 \leq k$$

$$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0$$

We assume that the resource constraint in the brain is binding, that is:

$$\varphi_1 + \varphi_2 + \varphi_3 \geq k$$

We take the following simple quadratic benefit function to illustrate the solution:

$$\vartheta_s(y_s; \varphi_s) = -\alpha_s (y_s - \varphi_s)^2 \tag{2.12}$$

The interior solution is:

$$y_s = \varphi_s - \frac{\frac{1}{\alpha_s}}{\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3}\right)} [\varphi_1 + \varphi_2 + \varphi_3 - k] \qquad for\ s \in \{1, 2, 3\} \tag{2.13}$$

For Task 1 and Task 2, plugging (2.13) in (2.10) and (2.11) leads to:

$$m_1 = \frac{\varphi_1 - \frac{\frac{1}{\alpha_1}}{\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3}\right)} [\varphi_1 + \varphi_2 + \varphi_3 - k]}{\varphi_1} \tag{2.14}$$

$$m_2 = \frac{\varphi_2 - \frac{\frac{1}{\alpha_2}}{\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3}\right)} [\varphi_1 + \varphi_2 + \varphi_3 - k]}{\varphi_2} \tag{2.15}$$

It is clear that more resources are allocated to a task if task importance, α_s, or the resources needed to successfully complete it (task complexity), φ_s, go up.

From (2.7) (the conditional expectations notation is suppressed):

$$E'\left(\pi_{t+1}^i\right) = E\left(\pi_{t+1}^i\right) + (1 - m_1)\left(\sum_{l \in S} v_l E\left(\pi_{t+1}^l\right) - E\left(\pi_{t+1}^i\right)\right) \tag{2.16}$$

Denoting the P/E ratio or earnings multiplier of firm i (inclusive of dividends) with c_i and using c_l to denote the P/E ratio of firm l:

$$E'\left(e_{t+1}^i\right) = E\left(e_{t+1}^i\right) + (1 - m_1)\left(\sum_{l \in S} v_l E\left(e_{t+1}^l\right)\frac{c_i}{c_l} - E\left(e_{t+1}^i\right)\right)$$

where $e_{t+1}^i = n_i^*\left(P_{t+1+}^i d_{t+1}^i\right)$ is the market value of total equity of i with n_i^* being the number of shares of firm i outstanding. It follows that:

$$\begin{aligned} E'\left(P_{t+1}^i + d_{t+1}^i\right) & \\ &= E\left(P_{t+1}^i + d_{t+1}^i\right) \\ &+ (1 - m_1)\left(\sum_{l \in S} v_l E\left(P_{t+1}^l + d_{t+1}^l\right)\frac{n_l^* c_i}{n_i^* c_l} - E\left(P_{t+1}^i + d_{t+1}^i\right)\right) \end{aligned} \tag{2.17}$$

Similarly, for risk:

$$Cov'\left(P_{t+1}^{i}+d_{t+1}^{i},M_{t+1}\right)$$

$$= Cov\left(P_{t+1}^{i}+d_{t+1}^{i},M_{t+1}\right)$$

$$+(1-m_2)\left(\sum_{l\in S} v_l Cov\left(P_{t+1}^{l}+d_{t+1}^{l},M_{t+1}\right)\frac{n_l^* c_i}{n_i^* c_l}\right.$$

$$\left.-Cov\left(P_{t+1}^{i}+d_{t+1}^{i},M_{t+1}\right)\right) \qquad (2.18)$$

where the aggregate market payoff, $X_{t+1}^{M}=n_1^*(P_{t+1}^{1}+d_{t+1}^{1})+n_2^*(P_{t+1}^{2}+d_{t+1}^{2})+\cdots\cdots\cdots$ $+n_Z^*(P_{t+1}^{Z}+d_{t+1}^{Z})$, with m_1 and m_2 given in (2.14) and (2.15) respectively.

2.2 Generalized Asset Pricing Model

Forecasting cashflow levels and the risk of cashflows are considered separate tasks in the brain. That is. These tasks are assigned to distinct systems of neurons. Each system demands resources which are allocated by the CES. The optimal resource allocation across the tasks depends on relative task importance and task complexity. The brain solves this problem as well as the problem of allocating finite wealth across assets (expected utility maximization). The general solution to the resource allocation in the brain problem is discussed in the last section. Here, we use this solution as an input in the expected utility maximization problem facing the representative investor.

Given the focus of investors on earnings news (Basu et al 2013) and the complexity of earnings or cashflow forecasting, we assume that importance assigned to the cashflow forecast by the CES is larger and more resources are needed for successful completion of the cashflow forecasting task. That is, $\alpha_1>\alpha_2$ and $\varphi_1>\varphi_2$. From (2.14) and (2.15), it follows that this increases the brain resources allocated to cashflow forecast and reduces the resources allocated to the risk forecast. Based on such considerations, we posit that typically $m_1 \sim 1$ and $m_2<1$ in the real world. We analyse this case here. For completeness, the discussion of the general case where both m_1 and m_2 are less than 1 is presented in the appendix.

Substituting (2.18) in (2.3) (and setting $c_i \sim c_l$, that is, firms in the same cluster have similar P/E ratios) leads to:

$$p_{it} = \frac{E_t\left[P_{t+1}^i + d_{t+1}^i\right]}{R_F} + Cov\left(P_{t+1}^i + d_{t+1}^i, M_{t+1}\right) + (1 - m_2)\left(\sum_{l \in S} v_l Cov\left(P_{t+1}^l + d_{t+1}^l, M_{t+1}\right)\frac{n_l^*}{n_i^*} - Cov\left(P_{t+1}^i + d_{t+1}^i, M_{t+1}\right)\right) \quad (2.19)$$

Multiplying (2.19) with n_i^*, adding across all stocks, and assuming that there are Q clusters in the market with L firms in each cluster:

$$p_{Mt} = \frac{E_t[X_{Mt+1}]}{R_F} + Cov(X_{Mt+1}, M_{t+1}) + (1 - m_2)\left[\sum_Q \sum_L \left(\sum_{l \in S} v_l Cov\left(P_{t+1}^l + d_{t+1}^l, M_{t+1}\right) n_l^* - Cov\left(P_{t+1}^i + d_{t+1}^i, M_{t+1}\right) n_i^*\right)\right] \quad (2.20)$$

where the aggregate market payoff is $X_{t+1}^M = n_1^*(P_{t+1}^1 + d_{t+1}^1) + n_2^*(P_{t+1}^2 + d_{t+1}^2) + \cdots\cdots\cdots + n_M^*(P_{t+1}^S + d_{t+1}^S)$

Re-arranging (2.20), and simplifying by setting all correlations with the SDF to -1[11],

$$\frac{E[R_{t+1}^M] - R_F}{\sigma(R_{t+1}^M)} = \frac{\sigma(M_{t+1})}{E(M_{t+1})} \times f \quad (2.21)$$

where $E[R_{t+1}^M]$ is the expected return on the aggregate market and

[11] This assumption simplifies the math considerably without any implications for subsequent analysis. For completeness, the model is derived with correlation coefficients allowed to be different than -1 in the appendix.

$$f = \left[1 + (1 - m_2)\left[\sum_Q \sum_L \left(\frac{\sum_{l \in S} v_l \sigma\left(R_{t+1}^l\right) w_l}{\sigma(R_{t+1}^M)} - \frac{\sigma\left(R_{t+1}^i\right) w_i}{\sigma(R_{t+1}^M)}\right)\right]\right] \qquad (2.21a)$$

$\sigma\left(R_{t+1}^l\right), \sigma\left(R_{t+1}^i\right), \sigma(R_{t+1}^M)$ are return standard deviations of l, i, and aggregate market respectively, $w_l = \frac{n_l^* p_{lt}}{p_{Mt}}$ (weight of firm l in the aggregate market portfolio), and $w_i = \frac{n_i^* p_{it}}{p_{Mt}}$ (weight of firm i in the aggregate market portfolio). Recall that v_l is the importance-weight assigned to firm l in the schema of that particular cluster.

(2.21) differs from the standard Sharpe-ratio expression due to the appearance of a multiplicative term f. This multiplicative term converges to 1 if the resource constraint in the brain does not bind, that is, when $m_2 = 1$.

Investor and analyst attention is highly asymmetric with most of the time spent on high market capitalization firms (Fang and Peress 2009). This asymmetry implies that a schema of the cluster is likely to be weighted towards such firms. That is, in a cluster of similar firms, the importance-weights, v_l, are larger for large market-cap firms when compared with small market-cap firms. With this in mind, we assume that v_l scales with w_l. For simplicity, we set $v_l = \frac{w_l}{\sum_{l \in S} w_l}$. It follows that $\sum_L \left(\frac{\sum_{l \in S} v_l \sigma(R_{t+1}^l) w_l}{\sigma(R_{t+1}^M)} - \frac{\sigma(R_{t+1}^i) w_i}{\sigma(R_{t+1}^M)}\right) > 0$. This implies that $f > 1$ if $m_2 < 1$. Proposition 1 follows.

Proposition 1 (Sharpe-Ratio Upper Bound) ***When the brain resources allocated to risk forecasting are less than the resources needed for successful task completion, the upper-bound on the equity Sharpe-ratio rises by a factor f which is greater than 1. The factor f is given as follows:***

$$\boldsymbol{f = \left[1 + (1 - m_2)\left[\sum_Q \sum_L \left(\frac{\sum_{l \in S} v_l \sigma\left(R_{t+1}^l\right) w_l}{\sigma(R_{t+1}^M)} - \frac{\sigma\left(R_{t+1}^i\right) w_i}{\sigma(R_{t+1}^M)}\right)\right]\right]}$$

Corollary 1.1 ***If the resource constraint in the brain does not bind, that is, if $m_2 = 1$, then the standard Sharpe-ratio upper bound is recovered.***

2.3 The Equity Premium Puzzle

How large is the multiplicative term f? The multiplicative term f in (2.21) is expected to be substantially larger than 1.

To get an idea about how large f is expected to be, consider the technology companies included in the ASX-200 (Australian Stock Market) index over a period ranging from 6th November 2019 to 6th November 2020. A total of 7 such technology companies have data available (Yahoo Finance) over the period considered. We calculate standard deviation of daily returns for each firm over this period and annualize them by multiplying with $\sqrt{250}$. We calculate market value weights for each firm as on Nov. 6, 2020. We assume that these firms belong to the same cluster with the importance-weight, v_l, increasing with the weight in the aggregate market in the following way: $v_l = \frac{w_l}{\sum_l w_l}$. That is, we set the importance-weight of a firm to be equal to the weight of the firm in the aggregate market divided by the weight of the cluster in the aggregate market. Table 1 shows return standard deviations, weights, and importance-weights of firms in the cluster.

We take return standard deviation of the aggregate market, $\sigma(R_{t+1}^M)$, to be 0.16, which is standard in much of the literature (Cochrane 2017). With these values:

$$\sum_{7}\left(\frac{\sum_{l\in S} v_l \sigma\left(R_{t+1}^l\right) w_l}{\sigma(R_{t+1}^M)} - \frac{\sigma\left(R_{t+1}^i\right) w_i}{\sigma(R_{t+1}^M)}\right) = 0.007$$

There are roughly 2100 firms listed in the Australian Stock Exchange, so with a cluster size of 7 firms, there are approximately 300 clusters. It follows that:

$$\sum_{300}\sum_{7}\left(\frac{\sum_{l\in S} v_l \sigma\left(R_{t+1}^l\right) w_l}{\sigma(R_{t+1}^M)} - \frac{\sigma\left(R_{t+1}^i\right) w_i}{\sigma(R_{t+1}^M)}\right) = 2.1$$

Setting $m_2 = 0.1$, it follows that:

$$f = \left[1 + (1-0.1)\left[\sum_{300}\sum_{7}\left(\frac{\sum_{l\in S} v_l \sigma\left(R_{t+1}^l\right) w_l}{\sigma(R_{t+1}^M)} - \frac{\sigma\left(R_{t+1}^i\right) w_i}{\sigma(R_{t+1}^M)}\right)\right]\right] = 2.9$$

Table 1

	Weight	Importance -Weight	Standard Deviation of Returns
Computershare Limited	0.0035	0.23	42.60%
NEXTDC Limited	0.0031	0.20	40.20%
Carsales.com Limited	0.0027	0.17	44.60%
Altium Limited	0.0025	0.16	41.80%
Technology One Limited	0.0015	0.09	38.40%
Link Adminstration Holdings Limited	0.0013	0.08	53.80%
IRESS Limited	0.0010	0.06	35.60%
Total Cluster Weight	0.0155	1	

Table 1: For technology companies included in the ASX 200 index, standard deviations of daily returns are calculated (and annualized by multiplying with $\sqrt{250}$) based on Yahoo Finance data for the period ranging from Nov. 6, 2019 to Nov. 6, 2020. Market value weights are calculated by dividing the total market-cap of the firm with the aggregate market capitalization as on Nov. 6, 2020. Importance-Weights are calculated by dividing the firm weight with the total cluster weight.

Historical equity premium has been between 4% to 8% with a standard deviation of around 16% on average. Using 6% as the equity premium, the left-hand-side of (2.21) is 0.375. Assuming power utility, and as standard practice, assuming lognormal consumption growth, it follows that:

$$\frac{\sigma(M_{t+1})}{E[M_{t+1}]} \approx \gamma\sigma(\Delta lnc)$$

where γ is the coefficient of risk-aversion.

In the post war data, aggregate consumption growth has been around 2%. Plugging these in (2.21):

$$0.375 = \gamma 0.02 f \tag{2.22}$$

Plugging $f = 2.9$, leads to:

$$\gamma = 6.47$$

In the above example, we have been quite conservative in estimating importance-weights with each firm in the cluster contributing to the schema based on its market cap. Instead, if we assume that only the top two firms by market-cap contribute to the schema of the cluster at 50% each, then $f = 6.14$, which implies that $\gamma = 3.05$. In other words, with reasonable assumptions regarding bigger contributions of larger market-cap firms to the cluster schema, f is substantially larger than 1, which lowers the risk-aversion needed to reconcile data with asset pricing theory.

2.4 The Size and Value Effects

Re-arranging (2.19) so that the Sharpe-ratio of stock i is on L.H.S:

$$\frac{E(R_{t+1}^i) - R_F}{\sigma(R_{t+1}^i)} = -\rho_i \frac{\sigma(M_{t+1})}{E[M_{t+1}]}\left\{1 + (1 - m_2)\left(\frac{\sum_{l \in S} v_l \sigma(R_{t+1}^l) w_l \rho_l}{\sigma(R_{t+1}^i) w_i \rho_i} - 1\right)\right\} \quad (2.23)$$

where w_l, w_i are market capitalizations of l and i respectively, and ρ_l, ρ_i are correlations of l and i with the SDF.

It is immediately clear from (2.23) that small market-cap firms have larger Sharpe-ratios as w_i appears in the denominator of R.H.S. Proposition 2 follows.

Proposition 2 (The Size Effect) ***Small market-cap firms have larger Sharpe-Ratios than large market-cap firms if the resource constraint in the brain binds.***

Proof.

From (2.23):

$$\frac{\partial(Sharpe\ Ratio)}{\partial w_i} = \rho_i \frac{\sigma(M_{t+1})}{E[M_{t+1}]}\left\{(1 - m_2)\left(\frac{\sum_{l \in S} v_l \sigma(R_{t+1}^l) w_l \rho_l}{\sigma(R_{t+1}^i) w_i^2{}_i \rho_i}\right)\right\}$$

The above is negative if the correlation of returns with the SDF, ρ_i, is negative as commonly assumed in the literature

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Just like the cross-sectional size effect, a cross-sectional value effect also arises from (2.23). Consider two stocks with identical fundamentals except that they have different schemas, S and S'. Assume that $\sum_{l \in S} v_l \sigma(R_{t+1}^l) w_l \rho_l > \sum_{l \in S'} v_l \sigma(R_{t+1}^l) w_l \rho_l$. It follows that stock with schema S is cheaper than stock with schema S' and has a higher Sharpe-ratio. Proposition 3 follows.

Proposition 3 (The Value Effect) ***A stock with a schema having higher risk is cheaper and has a higher Sharpe-ratio than an identical stock with a schema having lower risk.***

It matters which cluster a firm belongs to. Two firms could be very similar expect for the fact that one belongs to a cluster with higher risk. Such a stock is the value stock in this framework.

2.5 Countercyclical Equity Premia

Empirical evidence shows that the equity risk premium is countercyclical. For example, Harvey (1989) showed that US equity risk premia are higher at business cycle troughs than they are at peaks. Similar results are reported in Bekaert and Harvey (1995), He, Kan, Ng and Zhang (1996) and Li (2001) among others.

Crouzet and Mehrotra (2020) show that large firms are less sensitive to the business cycle fluctuations than small firms. In particular, they find that a 1% drop in GDP is associated with a 2.5% drop in sales at the top 1% of firms and a 3.1% drop in sales in the bottom 99% of firms. With the notion of scarcity pushed inside the human brain, a key implication of large firms being less cyclically sensitive is the countercyclical Sharpe-ratio of the aggregate market as explained below.

As large firms are expected to dominate (or at the very least have a bigger impact on) schemas in their respective clusters, less cyclical sensitivity of such firms makes the schema-generated starting points less cyclically sensitive as well when compared with the actual earnings of a typical firm. That is, $\sum_{l\in S} v_l Cov\left(\left(\pi_{t+1}^l, M_{t+1}\right)|I\right)$ is less cyclically sensitive than $Cov\left(\left(\pi_{t+1}^i, M_{t+1}\right)|I\right)$. So, the difference between $\sum_{l\in S} v_l Cov\left(\left(\pi_{t+1}^l, M_{t+1}\right)|I\right)$ and $Cov\left(\left(\pi_{t+1}^i, M_{t+1}\right)|I\right)$ is largest at the bottom (trough) of a recession and smallest at the top of an expansion. In other words, the schema-generated forecast or starting point is most accurate at the top of an expansion (requiring less brain resources to fully adjust) and least accurate at the bottom of a recession (requiring more brain resources to fully adjust). This makes $\left\{\sum_{l\in S} v_l Cov\left(\left(\pi_{t+1}^{l}, M_{t+1}\right)|I\right) - Cov\left(\left(\pi_{t+1}^{i}, M_{t+1}\right)|I\right)\right\}$ counter-cyclical. It is straightforward to see that if $\left\{\sum_{l\in S} v_l Cov\left(\left(\pi_{t+1}^{l}, M_{t+1}\right)|I\right) - Cov\left(\left(\pi_{t+1}^{i}, M_{t+1}\right)|I\right)\right\}$ is counter-cyclical then f is counter-cyclical as well. Proposition 4 directly follows.

Proposition 4 (Countercyclical Equity Premia) ***Sharpe-Ratio of the aggregate market is countercyclical.***

3. Conclusions and Discussion

Human brain is the ultimate scarce, efficient, and rational resource that first must optimize on itself before optimizing on the resources available in the external world. In this article, we adjust the standard asset pricing theory for optimal resource allocation of scarce resources in the brain. Intriguingly, a unified explanation for a diverse range of asset pricing anomalies (associated with the standard framework) emerges in the enriched framework.

While there may be other theories that can individually explain the equity premium puzzle (to an extent) and other effects by enriching the framework in a certain way, the model developed in this paper addresses all such “anomalies” within a single framework grounded in neuroscience and the actual functioning processes of the human brain.

One of the most striking implications of pushing the notion of scarcity inside the human brain is that the resulting enrichment is quite simple yet very powerful in its ability to explain a diverse range of phenomena. By pushing the notion of scarcity inside the human brain, Siddiqi and Murphy (2020) propose an enrichment of the standard CAPM framework. In this article, we use the same approach to propose an enrichment of the standard consumption-based asset pricing model. A natural task for future research is to see what other insights can be discovered by pushing this approach further and in other areas such as option pricing.

References

Alonso, Brocas, and Carrillo (2014), "Resource allocation in the brain", *Review of Economic Studies*, Vol. 81, pp. 501-534.

Anderson RC, Pearson PD. (1984), "A schema-theoretic view of basic processes in reading comprehension", In *Handbook of reading research* (ed. PD Pearson, R Barr, ML Kamil). Lawrence Erlbaum, Mahwah, NJ. American Psychiatric Association (2013). Diagnostic and statistical manual of mental disorders. 5th. Washington, DC.

Anderson, J. R. (2000). Cognitive Psychology and Its Implications (5th ed.). New York, NY. Worth Publishers.

Bartlett FC. (1932), "*Remembering: a study in experimental and social psychology",* Cambridge University Press, Cambridge.

Bossaerts, P. (2009), "What decision neuroscience teaches us about financial decision making", *Annual Review of Financial Economics,* pp. 383-404.

Bransford JD, Johnson MK. (1972), "Contextual prerequisites for understanding—some investigations of comprehension and recall", *Journal of Verbal Learning and Verbal Behavior,* Vol. 11, pp. 717–726

Basu, Duong, Markov and Tan (2013), "How Important are Earnings Announcements as an Information Source", *European Accounting Review*, Vol. 22, pp. 221-256.

Brod G, Lindenberger U, Werkle-Bergner M, Shing YL. (2015), "Differences in the neural signature of remembering schema-congruent and schema-incongruent events", *Neuroimage,* Vol. 117, pp. 358–366

Crouzet, N., and Mehrotra, N. (2020), "Small and Large Firms over the Business Cycle", *American Economic Review,* Vol. 110, No. 10, pp. 3549-3601.

Fang and Peress (2009), "Media coverage and the cross section of stock returns", *Journal of Finance*, Vol. 64, pp. 2023-2052.

Frazzini and Pedersen (2014), "Betting against beta", *Journal of Financial Economics,* Vol. 114, pp. 1-25.

Ghosh VE, Gilboa A. (2014), "What is a memory schema? A historical perspective on current neuroscience literature", *Neuropsychologia*, Vol. 53, pp. 104–114

Ghosh VE, Moscovitch M, Melo Colella B, Gilboa A. (2014), "Schema representation in patients with ventromedial PFC lesions", *Journal of Neuroscience,* Vol. 34, pp. 12057–12070

Gilboa, A., and Marlatte, H. (2017), “Neurobiology of schema and schema mediated memory”, *Trends in Cognitive Science*, Vol. 21, pp. 618-631.

Hampson, P. J. & Morris, P. E. (1996) Understanding Cognition. Cambridge, MA. Blackwell Publishers.

Hare TA, O’Doherty J, Camerer CF, Schultz W, Rangel A. (2008), “Dissociating the role of the orbitofrontal cortex and the striatum in the computation of goal values and prediction errors”, *Journal of Neuroscience,* Vol. 28, pp. 5623–30

Lai, Saridakis, Blackburn, Johnstone (2016), “Are HR responses of small firms different from large firms in times of recessions?”, *Journal of Business Venturing,* Vol. 31, Issue 1, pp. 113-131.

Levy, D., Glimcher, P. (2012), “The root of all value: a neural common currency for choice”, *Current Opinions in Neurobiology,* Vol. 22, pp. 1027-1038.

Lin, W., Horner, A., Bisby, J., and Burgess, N. (2015), “Medial prefrontal cortex: Adding value to imagined scenarios”, *Journal of Cognitive Neuroscience,* Vol. 27, Issue 10, pp. 1957-1967.

Preuschoff K, Bossaerts P, Quartz S. (2006), “Neural differentiation of expected reward and risk in human subcortical structures”, *Neuron*, Vol. 51, pp. 381–90.

Ohki and Takei (2018), “Neural mechanisms of mental schema: a triplet of delta, low beta/spindle and ripple oscillations”, *European Journal of Neuroscience,* Vol. 48, pp. 2416-2430.

Pankin, J. (2013), “Schema theory and concept formation”, *Mimeo,* MIT. Available at: http://web.mit.edu/pankin/www/Schema_Theory_and_Concept_Formation.pdf

Patry, M. B., and Horn, E. M. (2019), “Schema development in individuals with Autism: A review of the literature”, *Review Journal of Autism and Developmental Disorders,* Vol. 6, pp. 339-355.

Perfetti, Charles & Liu, Ying & Fiez, Julie & Taylor, Jessica & Bolger, Donald & Hai, Li. (2007), “Reading in two writing systems: Accommodation and assimilation of the brain's reading network”, *Bilingualism: Language and Cognition*, Vol. 10, pp, 131-146.

Peters, J., and Buchel, C. (2007), “Neural representations of subjective reward value”, *Behavioural Brain Research,* Vol. 213, pp. 135-141.

Piaget, J. (1936). *Origins of intelligence in the child.* London: Routledge & Kegan Paul.

Plassmann H, O’Doherty J, Rangel A. (2007), “Orbitofrontal cortex encodes willingness to pay in everyday economic transactions”, *Journal of Neuroscience,* Vol. 27, pp. 9984–88

Rangel, A., and Hare, T. (2010), “Neural computations associated with goal-directed choice”, *Current Opinions in Neurobiology,* Vol. 20, pp. 162-170.

Rushworth, M. F., and Behrens, T. E. (2008), “Choice, uncertainty and value in prefrontal and cingulate cortex”, *Nature Neuroscience*, Vol. 11, pp. 389–397.

Sahin, Cororaton, Kitao, and Laiu (2011), “Why small business were hit harder by the recent recession,” *Current Issues in Economics and Finance,* Federal Reserve Bank of New York, Vol. 17.

Sescousse, G., Caldu, X., Segura, B., and Dreher, J. (2013), “Processing of primary and secondary rewards: a quantitative meta-analysis and review of human functional neuroimaging studies”, *Neuroscience & Biobehavioral Reviews,* Vol. 37, pp. 681-696.

Siddiqi, H., and Murphy, J. A. (2020), “Resource Allocation in the Brain and the Capital Asset Pricing Model”. Available at SSRN: https://ssrn.com/abstract=3591086 or http://dx.doi.org/10.2139/ssrn.3591086

Siddiqi, H. (2019), “Anchoring-Adjusted Option Pricing Models”, *Journal of Behavioral Finance,* Vol. 20, Issue 2, pp. 139-153.

Siddiqi, H. (2018), “Anchoring-Adjusted Capital Asset Pricing Model”, *Journal of Behavioral Finance,* Vol. 19, Issue 3, pp. 249-270.

Spalding, Jones, Duff, Tranel, and Warren (2015), “Investigating the Neural Correlates of Schemas: Ventromedial Prefrontal Cortex Is Necessary for Normal Schematic Influence on Memory”, *Journal of Neuroscience,* Vol. 35, pp. 15745-15751.

Sweegers, Coleman, van Poppel, Cox, and Talamini (2015), “Mental schemas hamper memory storage of goal-irrelevant information”, *Frontiers in Human Neuroscience*, Vol. 9, pp. 6-29.

Tse D, Takeuchi T, Kakeyama M, Kajii Y, Okuno H, Tohyama C, Bito H, Morris RG (2011), “Schema-dependent gene activation and memory encoding in neocortex”, *Science,* Vol. 333, pp. 891–895

Tse D, Langston RF, Kakeyama M, Bethus I, Spooner PA, Wood ER, Witter MP, Morris RGM (2007), “Schemas and memory consolidation”, *Science,* Vol. 316, pp. 76–82.

van Kesteren MT, and Meeter (2020), “How to Optimize Knowledge Construction in the Brain”, *Science of Learning,* 5, Article Number 5

van Kesteren MT, Ruiter DJ, Fernandez G, Henson RN (2012), “How schema and novelty augment memory formation”, *Trends in Neuroscience*, Vol. 35, pp. 211–219.

van Kesteren MTR, Rijpkema M, Ruiter DJ, Fernández G. (2010), “Retrieval of associative information congruent with prior knowledge is related to increased medial prefrontal activity and connectivity”, *Journal of Neuroscience*, Vol. 30, pp. 15888–15894

Wadsworth, B. J. (2004), “*Piaget's theory of cognitive and affective development: Foundations of constructivism”,* New York: Longman.

Wallis, J. D. (2007), “Orbitofrontal cortex and its contribution to decision-making”, *Annual Review of Neuroscience,* Vol. 30, pp. 31-56.

Appendix

Substituting (2.17) and (2.18) in (2.3) and re-arranging (while assuming that $c_i \sim c_l$, that is, firms in the same cluster have similar P/E ratios):

$$\frac{E[R_{t+1}^M] \times g - R_F}{\sigma(R_{t+1}^M)} = -\rho_M \frac{\sigma(M_{t+1})}{E(M_{t+1})} \times f \qquad (A1)$$

where:

ρ_M is the correlation between the aggregate market return and the SDF

$$g = \left[1 + (1 - m_1) \sum_Q \sum_L \left(\frac{\sum_{l \in S} v_l E\left(R_{t+1}^l\right) w_l}{E(R_{t+1}^M)} - \frac{E\left(R_{t+1}^i\right) w_i}{E(R_{t+1}^M)} \right) \right] \qquad (A2)$$

$$f = \left[1 + (1 - m_2) \left[\sum_Q \sum_L \left(\frac{\sum_{l \in S} v_l \sigma\left(R_{t+1}^l\right) w_l \rho_l}{\sigma(R_{t+1}^M) \rho_M} - \frac{\sigma\left(R_{t+1}^i\right) w_i \rho_i}{\sigma(R_{t+1}^M) \rho_M} \right) \right] \right] \qquad (A3)$$

ρ_l and ρ_i are correlations of firm l and firm i's returns with the SDF respectively.

(A1) converges to the standard Sharpe-ratio expression when the resource constraint in the brain does not bind, that is, when $m_1 = m_2 = 1$. Given that most of investor and analyst time is spent on cashflow or earnings forecast, the relative importance and complexity of the brain task of developing earnings forecast are larger. From (2.14) and (2.15), it follows that $m_1 > m_2$ with $m_1 \sim 1$. This is the case which is discussed in the main body of the article.