FORWARD BETA THAT INCLUDES MARKET AND STOCK FUNDAMENTALS

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ABSTRACT

Beta is an important variable used as an input in determining a discount rate (or a required return) to calculate the fair value of equity. Efforts have been made to improve the quality of Beta's estimate beyond the typical approach of regressing past prices by including company fundamental factors. This paper looks at Beta as a function that finds equilibrium between outlook of company fundamentals and market dynamics; its derivation lies somewhere between the domains of equity analysts and market strategists. Sample of work includes both developed and developing markets.

KEY WORDS

Beta, CAMP, Discount Rate, WACC, Equity Risk Premium, Market Risk Premium, Valuation, Developing Market

INTRODUCTION

Required Returns on Equity or Weighted Average Cost of Capital (WACC), derived from the Capital Asset Pricing Model (CAPM), is widely used as a discount rate to discount streams of cashflows in order to arrive at an equity value. (This paper will restrict the discussions to equity asset class only.) Critically, every variable that goes into the valuation process is 'forward-looking', with the typical exception of Beta which generally is derived from historical stock prices.

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Results from a paper published Dr. Pablo Fernandez[i] that surveyed 2,500 professors show that most of the professors who use Beta make use of regressions, webs, database, and textbooks. Many of them also admit that these Betas are poorly measured and have many problems.

Another paper published by Jennifer Bender and Frank Nielsen from MSCI Research Insight [ii] introduced some fundamental factors which are further split into qualitative and quantitative factors. The former includes business model, competitive advantage, management quality and corporate governance while the latter includes historical and estimates of capital structure, revenues, earnings and cashflows. Another paper by B Rajesh Kumar and Manuel Fernandez[iii] introduced risks elements associated with earnings and cashflows through variance of these factors.

The major difference adopted by this paper is that the fundamental factors are defined from 2 perspectives namely, market-related fundamentals and stock-related fundamentals. Market-related fundamentals are associated with the shape of the government yield curve, risk appetite, including commodity prices or exchange rates in cases where markets are heavily weighted towards businesses that are linked to commodity and exchange rates movements.

Stock-related factors include relative earnings growth (to the index), long-term earnings growth, leverage, profitability, book value.

DISCUSSION

Beta can be described as a measure of a stock's relative performance to that of a market index. A stock with a Beta of 1 means it is expected to generate the same return as the index. A stock with a Beta of more than 1 means it is expected to generate higher (or lower) returns than the market index by the extent of its deviation from '1'.

Here we see that the interpretation of Beta is inherently subjective to the direction of the market that one takes. A stock with a Beta of 1.5x means the stock is expected to generate returns 1.5x higher than that of the index if the index goes up. Conversely, if the index goes down, the stock can be expected to underperform the index by 1.5x.

The most common method of calculating Beta is to divide the covariance of returns between the stock and index with the variance of the returns of the index. Mathematically, its formula can be written as:

Beta of a Stock = Covariance of Stock Return and Index Return / Variance of Index Return

Intuitively, the formula measures how strongly the two sets of returns are correlated to each other. This correlation is then divided by the variability of the returns of the market to get a per unit correlation to the variability of the index.

Beta can also be calculated using Excel spreadsheet through the use of the 'Slope' function and Regression Analysis.

Some of the weaknesses in the use of traditional Beta are identified here.

1. Backward Looking

A major problem with the current definition is that Beta is derived only from past prices and the past prices already reflected the prevailing market conditions and company outlook. This methodology implicitly assumes that the economic environment, monetary conditions, relative competitiveness against peers will remain the same in the future as in the past.

This point is better appreciated when we consider the entire process of valuation. When equity analysts (buy-side or sell-side) estimate the value of equity, they start with developing financial models to arrive at forecasts on Sales, Margins, Profits, Earnings Per Share (EPS), Cashflows etc. This process is time-consuming and demands considerable efforts and experience to crystallize their views about the industry, company operations and management strategy into a set of assumptions that best fit their views of the future world.

Once the company's bottom-up forecasts are made, analysts calculate the discount rate to arrive at a fair value of equity. This discount rate may be either *Weighted Average Cost of Capital (WACC)* or *Required Returns on Equity (Rqd. ROE)* depending on whether they want to use free cashflows or EPS in the numerator.

Both WACC and Rqd. ROE further require the following variables as inputs:

- *Long-Term Risk-Free Rate:* This is often proxied as the 10-Year Government Bond Yield (10Y GBY). Typically, the data can be read using current market yield which represents a 10-Year forward view of risk-free rate. In some cases, economists make their own forecasts.
- *Market Risk Premium (MRP):* This risk premium is demanded by equity investors when investing in the equity market (index). MRP is a measure of risk appetite and is often a forward-looking number commonly estimated by market strategists.
- *Cost of Debt:* This can be calculated from the current bond yields or forecasts can be made based on credit rating and credit spread. If the debts are not traded, the cost of debt for the latest year can be adjusted to reflect changes in policy rates.
- *Beta:* In most cases known, historical prices are used to calculate Beta.

All variables mentioned above reflect forward-looking views, except Beta. This considerably undermines the worthiness of valuation.

2. Asymmetric Interpretations

The value of Beta used in framing investment portfolio depends on the direction of the market that an investor takes. A Beta of more than 1 should be interpreted as positive to buy if one takes a view that the market will be rising. But if one believes the market will fall, stocks with Beta more than 1 should be avoided.

So higher or lower Beta must be interpreted within the context of market direction in order to arrive at an actionable conclusion. This makes interpretations of Beta subjective to an assumed outcome. Some have tried to apply 'Up Beta' or 'Down Beta'. However, to apply them we still need to have a view of future market direction in order to choose which Beta to use.

A potentially disruptive outcome is when market conditions during when Beta is calculated become different than when it is applied.

3. Lookback Period

Choice of the lookback period can lead to differences in the calculation of Beta. A short lookback period may reflect the most current conditions well but may not be appropriate to apply in the long term. On the other hand, a long lookback period may ignore short-term market cycles. In theory, the lookback period is supposed to cover a full industry cycle but mapping the industrial cycle to price cycle itself is not a precise science.

Most academic papers prefer to use a relatively long lookback period. From the author's experience, a rolling daily 12-month or 6-month is commonly adopted by sell-side analysts. This may appear short and periodicity too frequent but the arguments in their favor are:

1. Business cycles are becoming shorter. Product life cycles are getting shorter due to ease in the adoption of new technology, already established distribution channels, efficient logistics and increased competitiveness in every segment of the value chain. Shorter industry cycles call for shorter lookback periods. Figure 2 reproduces a chart from a well-known publication on valuation. It shows shorter product life cycles for newer products.

Figure 1: Newer industries have shorter life cycles.
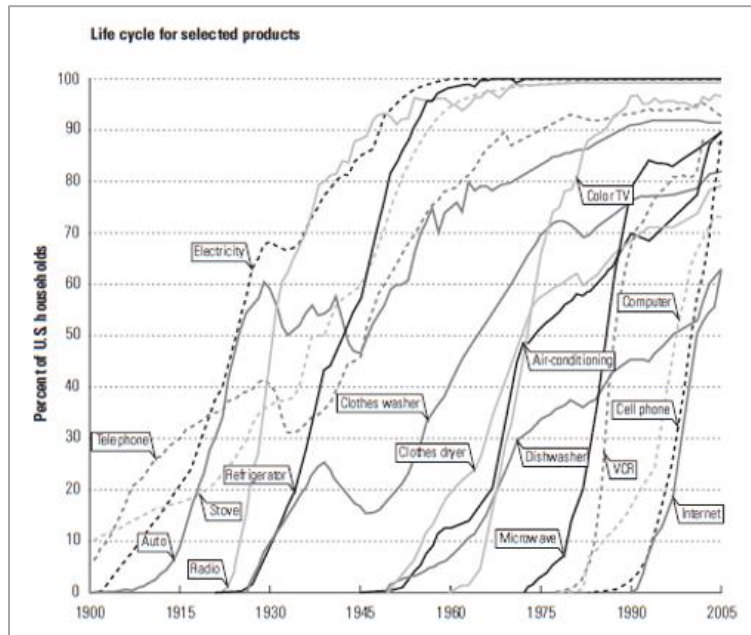
Source: Reproduced from the book: VALUATION MEASURING AND MANAGING THE VALUE OF COMPANIES, FIFTH EDITION, McKinsey & Company. It further quotes the source from W. Cox and R. Alm, 'You Are What You Spend". New York Times, February 1, 2008.

2. A paper (*[New Data Shed Light on Mutual Fund Time Horizons | CLS Blue Sky Blog (columbia.edu)](#)*) looking into the holding period of mutual funds based on Portfolio Trading Ratio (PRT) showed that PTR increased from 26% in 1945 to 45% in 1975 and to 79% between 2005-2015. A 100% PTR equals a holding period of 12 months; 79% theoretically puts the average holding period at 15 months.

3. Decline in transaction costs, ease of trading, wider diversity and accessibility of equity products and their derivatives all point to the down trending of vested period. According to a paper published by the World Economic Forum (*[Long-term investing: what are the reasons behind its decline? | World Economic Forum (weforum.org)](#)*), average holding of a stock in the New York Stock Exchange (NYSE) has declined from 8 years in the late 1950s to 5.5 months as of June 2020. Note that this is an average holding period for all types of investors computed from total trades. It is likely that much of that dramatic decline has been led by increased participation of retail investors and high-frequency traders.

4. Empirically, we see sell-side analysts' coverage moving from making annual forecasts to quarter forecasts, suggesting that milestones of investments are getting shorter. Quarterly earnings conference calls for large companies have often become prime-time content among mass media as well. Shortermism is in vogue – whether we like it or not.

With compensation for executives tied to stock prices and stock options, this trend will not be easily reversed.

4.   No-Man's Land

It is understandable that once economists and strategists have made estimates of 10Y GBY and MRP respectively, their analytical tasks are considered delivered. It also appears that equity analysts' major analytical tasks are considered done once they have completed financial forecasts as valuation is often seen as a mathematical process.

This leaves out Beta under-scrutinized as a parameter that finds the equilibrium point between market conditions and company fundamentals. This point is explained in the next section. The paper suggests that the role of equity analyst be expanded to cover Beta forecast too since Beta is a factor associated with the valuation of a stock.

Note that the author recognizes that the roles of economists, strategists and analysts simply do not stop once results of analysis are made. They also need to write papers, market their works, and continually monitor their views.

METHODOLOGY

The method proposed in this paper eliminates some weaknesses discussed above. Let us begin with the CAPM formula to calculate a discount rate.

Rqd. ROE   = Long-Term Risk-Free Rate + MRP x Beta

Substituting Long-Term Risk-Free Rate with 10Y GBY:

Rqd. ROE   = 10Y GBY + (MRP x Beta)

Rqd. ROE – 10Y GBY   = MRP x Beta

Equity Risk Premium (ERP)   = MRP x Beta

Beta   = ERP / MRP

                              = Function of ERP and MRP

Figure 2: Beta creates price discovery between market dynamics and company fundamentals.

Top-Down Macro Conditions:
Economists estimate LT Risk Free and Strategists estimate MRP. Together, we get Rqd. ROE for an equity index.

Bottom-Up Company Fundamentals:
Analysts take views on industry and company outlook. They make financial forecasts.

Beta

***Beta** finds the equilibrium point between top-down market dynamics and company fundamentals.*
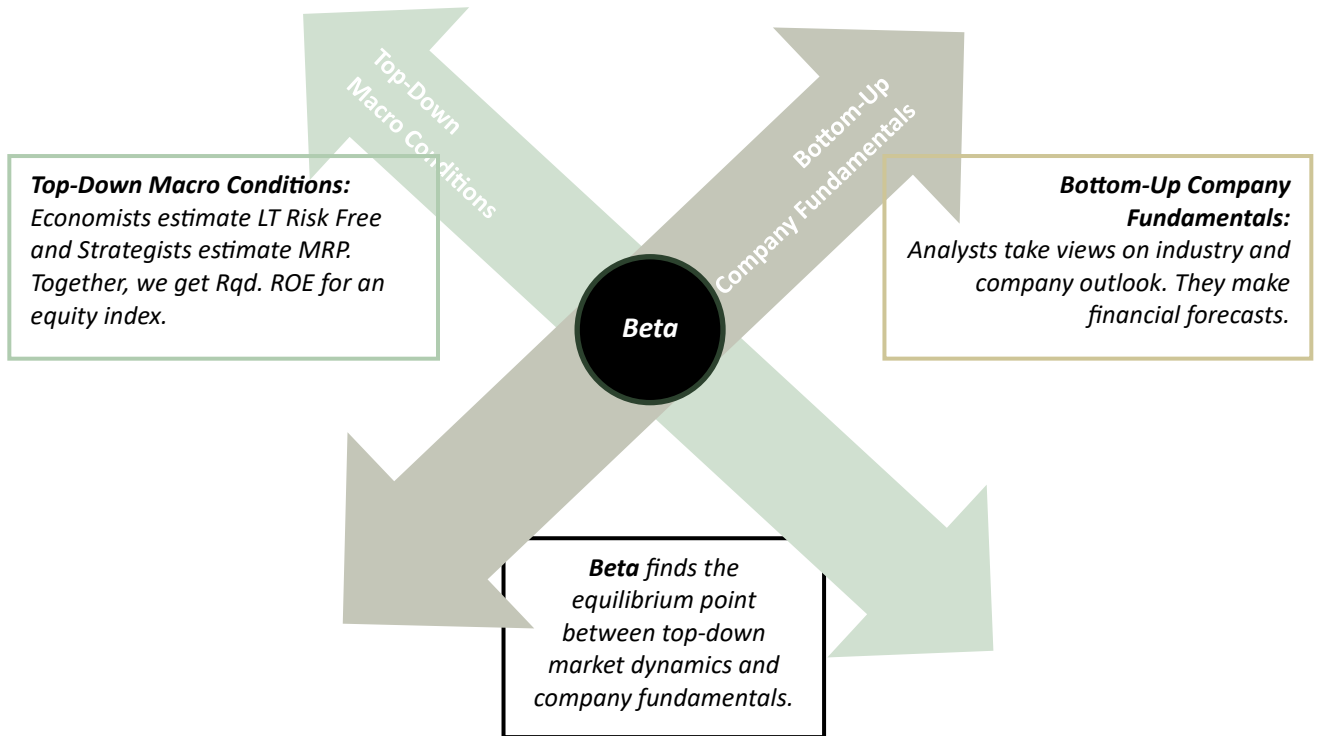
From the formula above, the Rqd. ROE can be observed as it is the realized Earnings Yields (EY) of a stock. EY is derived by dividing consensus earnings estimates with stock price. Subtracting EY with 10Y GBY results in ERP of the stock. MRP can be similarly calculated by replacing forward earnings consensus estimates of a stock with forward earnings consensus estimates of the index and replacing the stock price with the index. With ERP and MRP known, realized Beta can be calculated.

Viewing Beta as a function of ERP and MRP offers an intuitive insight into its drivers. Beta becomes a number that reflects the results of interaction between fundamental factors of a stock and fundamental factors of the market.

Next, we list these fundamental factors of a company and market (index) that can be independently observed and forecasted which can have influence on ERP and MRP. We have shortlisted the following.

Company Factors:

1. 12-Month Forward Relative EPS Growth (12M FWD Rel. EPS Growth). This refers to the expected EPS growth of the stock over the next 12-month period less that of the

expected EPS growth of the index over the next 12-month period. The more the deviation of stock EPS growth over that of the index, the greater is the potential to outperform or underperform the index.

2. 3-Year Forward EPS Growth (3Y FWD EPS Growth). This factor takes into account stocks that are expected to have strong growth in the longer term even though the near-term outlook does not look attractive. In cases where 3Y forward EPS are not available, 2Y estimates are used.

3. 12-Month Forward Book Value (12M FWD BV). This is to take into account stocks that do not have strong earnings growth or sectors where valuation is more driven from the angle of value.

4. 12-Month Forward ROE (12M FWD ROE). Slow growth stocks tend to trade based on their BVs and profitability on equity.

5. 12-Month Forward Net Debt Equity Ratio (12M FWD D/E). The degree of leverage can have an influence on stock prices.

Market Factors:

1. 10Y GBY. The 10Y Government Bond Yield is widely used as a benchmark for long-term risk-free rates. Its movement has a great impact on the valuation of financial assets.

2. 2Y GBY. The 2Y Government Bond Yield is included to take into account the near-term interest rate direction.

3. In some markets where the constituents' business is much driven by exports, we should include the exchange rate as a factor as well. We have used Euro/USD (USDEUR) and Thai Baht/USD (USDTHB) exchange rates as inputs for EU and Thailand markets.

4. In a market where the index is heavily weighted towards a commodity, we should also include that commodity as a fundamental factor. The Thai stock market has about 20% of its market capitalization exposed to Energy sector, so we have included West Texas Crude oil (WTI) price as a factor as well.

Once these factors are identified, their historical data points are used as independent variables in a multiple regression with realized Beta (calculated as ERP/MRP) as the dependent variable. Daily data going back 4 years from Jan-2019 to Dec-2022 are used to back-test. The choice of period selected is to ensure that:

1. the analysis covers the pre-Covid period, during Covid and post-Covid, so practically covering a business cycle.

2. Results from the back-test are used to estimate fair prices in the forward-test for the following 1 quarter period ending Mar-2023. This date has been chosen because it is the last date that forward-looking data is available for analysis in this paper.

3. Alternatively, back-testing up to June-2022 has been considered to allow a for longer period of forward testing. However, this option has not been selected because the period between Jul-2022 to Dec-2022 corresponded to the period where LT GBY across most countries rose sharply, which the author expects high levels of rates will prevail in the future.

Both historical prices and forward-looking estimates used are sourced from Bloomberg.

We will call Beta calculated from multiple regression as Fundamental Beta or Beta (f). This is to differentiate from the traditional approach of using historical prices which we will call Beta (p).

Figure 3: Steps to calculation Fundamental Beta or Beta (f)

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1. Calculate daily Realized ERP of a stock.
   - (12M FWD EPS Estimate / Stock Price) – 10Y GBY
2. Calculate daily Realized MRP of an index.
   - (12M FWD EPS Estimate of index / Index) – 10Y GBY
3. Calculate daily Realized Beta = ERP/MRP. This is the Realized Beta which will be used as a Dependent Variable in a multiple regression.
4. The 9 identified financial variables above are used as independent variables. The variables should be those that can be forecasted in order to enable forecast Beta to be calculated.
5. Run the multiple regression with the minimum threshold to accept at adjusted R2 of 75% and p-value of individual variables at less than 5%.
6. Using coefficients from the multiple regression, Beta (f) can be calculated.
7. As forecasts of independent variables are available, forecast Beta can be calculated.

Figure 4 shows an excel cut-out of inputs from the multiple regression analysis. The example shown is 'Microsoft Corporation'. Note that independent variable '12M FWD ROE' has been ignored as its p-value did not pass the minimum 5% test.

**Figure 4: Excel cut-out on regression analysis for 'Microsoft Corp.'**

	Independent Variables						Dependent Variable	Fundamental Beta
	12M D/E	12M FWD Rel. EPS Growth	3Y FWD EPS Growth	12M FWD BV	2Y UST Y	10Y UST Y	Realized Beta (ERP/MRP)	Beta (f)
2-Jan-19	-45.8%	10.5%	33.6%	13.7	2.47%	2.62%	0.49	0.50
3-Jan-19	-45.8%	10.6%	33.6%	13.7	2.38%	2.55%	0.52	0.50
4-Jan-19	-45.8%	10.9%	33.6%	13.7	2.49%	2.67%	0.49	0.49
7-Jan-19	-45.8%	11.3%	33.6%	13.8	2.54%	2.70%	0.49	0.49
8-Jan-19	-45.8%	11.5%	33.7%	13.8	2.59%	2.73%	0.49	0.49
22-Dec-22	-17.4%	9.7%	15.5%	34.0	4.27%	3.68%	0.29	0.22
23-Dec-22	-17.4%	9.7%	15.5%	34.0	4.32%	3.75%	0.27	0.21
26-Dec-22	-17.4%	9.8%	15.5%	34.1	4.32%	3.75%	0.27	0.21
27-Dec-22	-17.4%	9.8%	15.5%	34.1	4.37%	3.84%	0.25	0.20
28-Dec-22	-17.4%	9.9%	15.5%	34.1	4.35%	3.88%	0.25	0.20
29-Dec-22	-17.4%	10.0%	15.5%	34.2	4.36%	3.81%	0.23	0.20
30-Dec-22	-17.4%	9.9%	15.5%	34.2	4.43%	3.87%	0.22	0.20

Figure 5 shows the results from multiple regression analysis with regressed adjusted R2 at 0.91. Figure 6 shows the results of valuation of back-test from Jan-19 to Dec-22 and the forward-test from Jan-23 to Mar-23. Note that macro factors in the forward test are assumed to remain the same as the last day in Dec-22 but company factors during the forward test are based on Bloomberg consensus estimates.

Figure 5: Regression results for 'Microsoft Corp.'

SUMMARY OUTPUT

Regression Statistics				
Multiple R	0.956375945			
R Square	0.914654948			
Adjusted R Square	0.914160672			
Standard Error	0.034640248			
Observations	1043			

ANOVA

	df	SS	MS	F
Regression	6	13.32295895	2.220493159	1850.493
Residual	1036	1.243144888	0.001199947	
Total	1042	14.56610384		

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.571001392	0.044332579	35.43672497	8.2E-181
12M D/E	0.298727663	0.048745084	6.12836491	1.26E-09
12M FWD Rel. EPS Growth	0.024639509	0.016252689	1.516026578	0.129818
3Y FWD EPS Growth	-1.218771045	0.06743813	-18.07243242	1.13E-63
12M FWD BV	-0.026822517	0.0011178	-23.99582034	1.5E-101
2Y UST Y	3.399824656	0.531437195	6.397415697	2.39E-10
10Y UST Y	-9.475603211	0.560960987	-16.891733	9.95E-57

Figure 6: Fair Price of 'Microsoft Corp.' Back-Test and Forward-Test
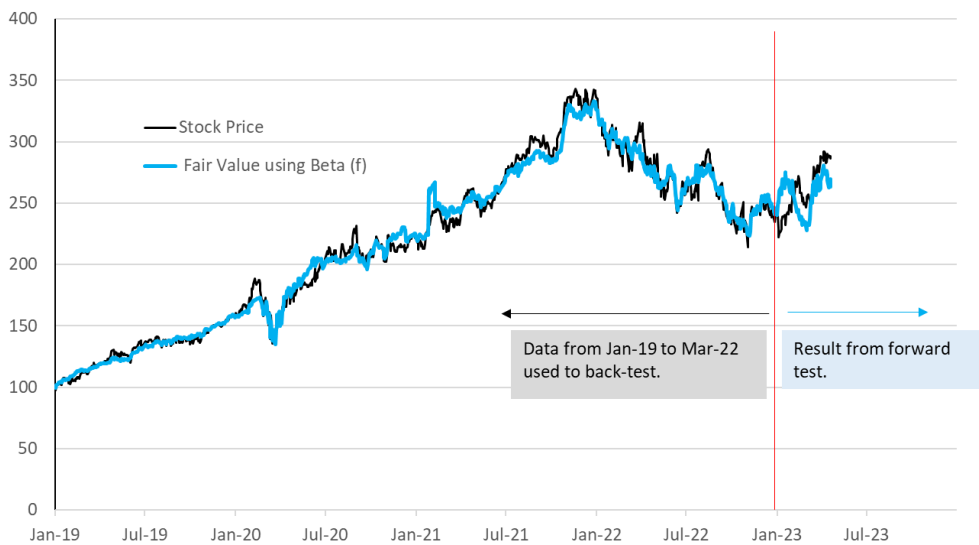
A total of 15 stocks have been used to demonstrate the results from using the proposed method. The 15 stocks chosen are based on geography, level of market development and their market capitalization including USA (S&P500) Eurozone (STOXX index) and Thailand-Asia (SET index). Thailand has been chosen because it is the home country of the author.

Chart 7: List of stocks included in the sample.

Stocks	Company Name	Short Name	Industry	Market Cap. (USD Bn)
Geography: US - S&P500 Index				
1	Microsoft Corp.	MSFT	Technology	2,500
2	Alphabet Inc.	GOOGL	Technology	1,600
3	Amazon.com Inc.	AMZN	Consumer Discretionary	1,300
4	NVIDIA Corp.	NVDA	Technology	1,000
5	Meta Corp.	META	Technology	770
Geography: EU - STOXX Index				
1	L'Oreal SA	OR	Consumer Goods	200
2	Vinci SA	DG	Construction & Services	80
3	Banco Santander SA	SAN	Financial Services	50
4	ASML Holding N.V.	ASML	Semi-Conductor	210
5	TotalEnergies SE	TTE	Oil & Gas Production	150
Geography: Thailand - SET Index				
1	PTT Plc.	PTT	Oil & Gas Production	25
2	Kasikorn Bank	KBANK	Financial Services	10
3	CP ALL Plc.	CPALL	Commerce	14
4	Advanced Info Service Plc.	ADVANC	Technology	18
5	Siam Cement Plc.	SCC	Construction Materials	1

Results of regression analysis for the 15 stocks tested are summarized in Figure 8.

Figure 8: Results from regression analysis for stocks sampled.

	Code	Adj. R2	12M FWD D/E	12M FWD Rel. EPS Growth	3Y FWD EPS Growth	12M FWD BV	12M FWD ROE	2Y GBY *	10Y GBY *	USDEUR Exchange Rate	Crude Oil (WTI)	USDTHB Exchange Rate
USA												
Microsoft Corp.	MSFT	0.914161	x	x	x	x	-	x	x	-	-	-
Alphabet Inc.	GOOGL	0.656318	x	x	x	x	-	x	x	x	-	-
Amazon.com Inc.	AMZN	0.938832	x	x	x	x	x	x	x	x	-	-
NVIDIA Corp.	NVDA	0.748413	x	x	x	x	x	x	x	x	-	-
Meta Corp.	META	0.844856	x	x	x	x	x	x	x	x	-	-
EU												
L'Oreal SA	OR	0.948488	x	x	x	x	x	x	x	x		
Air Liquid SA	AI	0.923432	x	x	x	x	x	x	x	x	x	-
Banco Santander SA	SAN	0.793358		x	x	x	x	x	x	x	x	-
ASML Holding N.V.	ASML	0.910791	x	x	x	x	x	x	x	x	x	-
TotalEnergies SE	TTE	0.901979	x	x	x	x	-	x	x	x	x	-
Thailand												
PTT Plc.	PTT	0.918032	x	x	x	x	x	x	x	-	x	x
Kasikorn Bank	KBANK	0.807391	-	x	x	x	x	x	x	-	x	-
CP ALL Plc.	CPALL	0.953779	x	x	x	x	x	x	x	-	x	x
Advanced Info Service Plc.	ADVANC	0.801618	-	x	x	x	x	x	x		x	x
Siam Cement Plc.	SCC	0.850787	x	x	x	x	x	x	x	-	x	x

* US Treasury Yields for US, Germany Government Bond Yields for EU and Thailand Government Bond Yields for Thailand

CONCLUSION

The provide validity to the approach proposed, it is necessary to compare the deviation of observed stock price with the estimated fair values derived from using the 2 sets of Betas. The following processes have been used.

- It is assumed that fair stock prices as of end Mar-23 need to be estimated. As data used to back-test runs upto Dec-22, the process to estimate fair price at end Mar-23 needs to occur at the end of Dec-22.
- Beta (f) is calculated using regression coefficients and company fundamental factors estimated for period Mar-23. Beta (p) calculated as of end Dec-22 is used to apply to forward period. Beta (p) is calculated based on a daily rolling 6-month basis.
- Data related to economy and market namely, 2Y GBY, 10Y GBY, Crude Oil price, Exchange Rates and MRP is assumed to remain the same from Dec-22 through Mar-23. This assumption will ensure that economic and market related data required as inputs to calculate Rqd. ROE for the 2 sets of Beta is the same.
- The 12M FWD EPS estimates used are the same in both cases.

Results to the validation in Figure 9 show that deviation from fair price estimates using Beta (f) are far lower than those of Beta (p) and with much less standard deviation (Figure 10).

Figure 9: Deviation predicted by Beta (f) is significantly lower than those of Beta (p)…

Stock	Predicted Fair Price for end Mar-23		Share Price	Deviation: Share Price to Predicted	
	Using Beta (f)	Using Beta (p)	as of Mar-23	Using Beta (f)	Using Beta (p)
MSFT	262	157	284.1	9%	81%
GOOGLE	101	88	100.9	0%	15%
AMAZON	115	40	102.0	-11%	154%
NVDA	162	58	273.8	69%	371%
META	111	150	207.8	88%	38%
OR	357	155	405.6	14%	162%
AI	138	79	152.7	11%	94%
SAN	4	7	3.5	-7%	-50%
ASML	554	N.A.	623.7	13%	N.A.
TTE	48	165	54.4	13%	-67%
PTT	34	57	31.3	-8%	-45%
KBANK	139	340	133.5	-4%	-61%
CPALL	73	34	62.3	-14%	84%
ADVANC	193	218	211.0	9%	-3%
SCC	304	486	315.0	4%	-35%
Average				12%	53%
Std. Deviation				28%	116%

* ASML's share price predicted by Beta (p) gives a negative number. With 10Y GBY at 2.74%, Beta (p) at 1.79% and MRP at 6.00%, the Rqd. ROE gives a negative number.

Figure 10: …with much less standard dispersion of errors.

Difference between Beta (p) and Beta (f)

Key differences between Beta-f and Beta-p can be summarized below.

<u>Figure 11: Summary of features between Beta (f) and Beta (p)</u>

Issue	Beta (f)	Beta (p)
Nature of data	Forward-looking factors	Historical share prices
Scope of data	Fundamental factors with choice depending on uniqueness of market and company	Only price data is used
Asymmetric outcome	Eliminated	Possible
Lookback period	Fundamental and economic factors easier to define as cycles	Price pattern less easily defined as cycles
Interpretation	Cannot be interpreted in the traditional way. Beta (f) is simply the best-fit ratio between company fundamentals with market and economic factors.	Implications of higher (lower) than 1.

LIMITATIONS

The proposed Beta (f) has some weaknesses.

1. Forward consensus estimates of companies are needed. This will limit the scope of analysis based on this approach to larger stocks that are well covered by sell-side analysts. There should also be a minimum number of analysts covering a stock to ensure the quality of estimates. A financial platform, which can be expensive, is also needed to get access to such data.

2. It is possible that Beta (f) becomes a negative number in cases where a stock consistently traded at very high valuation in the past due to expectation of high growth, resulting in negative ERP (where historical EY was consistently lower than 10Y GBY). A possible way out is to use Sales/Stock Price instead of EY. However, this option has not been tested.

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