Derivative warrant market and implied volatility

Evidence from Stock Exchange of Thailand

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Abstract

This project tested whether the Thai derivative warrant price is closer to the Black and Scholes pricing theory over the Thai derivative warrant market history. It was found that the implied volatility from the derivative warrant price is 66.15% on the average, while the subsequent realized volatility is 34.39% on the average. This finding can be concluded that the Thai derivative warrant price is overvalue and the prediction of the volatility by the issuers which can be seen from the implied volatility from the derivative warrant price is overestimate. From the regression model result, it also found that the implied volatility forecast error or the different between the implied volatility and the subsequent realized volatility does not reduce over the years, comparing to when Thai derivative warrant market starts in year 2009 and when it is more mature in year 2015. It means that the prediction of the volatility is not better over the years.

Moreover, it was found the reversion of the implied volatility forecast error from year to year. From the further study, it was found that the reversion is the result of the subsequent realized volatility movement, while the implied volatility is quite smooth over the years.

The project also found that even there are more derivative warrant issuers over the years. This higher competitive market leads neither the cheaper derivative warrant nor the higher effective Thai derivative warrant market which can be seen from the exceeding of the time-series average implied volatility over the time-series average realized volatility which does not reduce over the years. Moreover, there is also the reversion of this exceeding.

Moreover, it was also found that the higher demand of the call derivative warrant affects the change in implied volatility or the derivative warrant pricing which can be seen
from the lower negative correlated between the implied volatility and the stock price over the years which can be conclude that the call derivative warrant pricing is higher when the stock price increase comparing year by year. However, this effect does not happen with the put derivative warrant and the surprising is that the change in put derivative warrant implied volatility positively correlated the change in stock price meaning that the put derivative warrant implied volatility is lower when the stock price decreases.
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**Introduction**

**Derivative warrant**

Derivative warrants are derivative securities that have the same contractual characteristics as options. Derivative warrants are the rights of buyers to buy or sell the underlying securities in the future at pre-determined price, quantity and period as specified by the issuers. Derivative warrants are issued by the third parties, which are not related to the listed company issuing the underlying securities. Derivative warrant issuers usually are the financial institutions. Derivative warrant has many common names such as covered warrant, equity linked warrant and structured warrant which is used differently in each country. In Thai market, it is called derivative warrant.

The features of derivative warrant are as followed:

1. **Calls and puts**: Derivative warrant has two basic forms which are a call and a put.
   
   Taking a long position in a call means the buyer has the right to buy the underlying at pre-determined price before or at expiration date, while taking a long position in a put means the buyer has the right to sell.

2. **Exercise style**: Derivative warrant can be either American style or European style.

   American-style derivative warrant can be exercise any time before expiration date, while European-style derivative warrant can be exercise only at expiration date. Thai derivative warrant is European style.

3. **Exercise price or strike price**: Predetermined price that the buyer has to pay (call) for buying the underlying asset or receive (put) for selling the underlying asset when exercise the derivative warrant.
4. Expiration date or maturity: Predetermined date that the derivative warrant will expire. It is the last day that can exercise American-style derivative warrant, while it is the only day that can exercise the derivative warrant for European-style derivative warrant.

5. Conversion ratio: This is the feature that makes derivative warrant different from the option. The conversion ratio is the number of derivative warrant required to gain exposure to one unit of the underlying asset.

6. Settlement: Derivative warrant is cash settlement, so there is no dilution effect on underlying asset. The return for the call derivative warrant can be calculated by using this equation:

\[
\frac{\max(0, \text{underlying price} - \text{strike price})}{\text{conversion ratio}}
\]

The return for the put derivative warrant can be calculated by using this equation:

\[
\frac{\max(0, \text{strike price} - \text{underlying price})}{\text{conversion ratio}}
\]

7. Gearing or leverage: This feature is the foremost benefit of the derivative warrant. It can be calculated by dividing the underlying asset price by the derivative warrant price adjusted the conversion ratio to be 1:1. Gearing or leverage measures the amount of additional exposure gained by investing in derivative warrants rather than directly in the underlying asset. On the contrary, leverage also means that investors may have higher loss as well. However, the maximum loss is capped at the total outlay on derivative warrant.

8. Others:

8.1 Derivative warrant has no dividend.
8.2 Derivative warrant has no shareholder’s right.

8.3 Derivative warrant cannot be short.

8.4 Derivative warrant has no margin.

8.5 Derivative warrant is traded in exchange.

Thai derivative warrant market

Derivative warrant began trading in Thailand since 2009. Derivative warrant was traded in the stock exchange of Thailand (SET). In 2009, there were only 3 derivative warrants issued in that year which the first three underlying of those derivative warrants were PTT, KBANK, and PTTEP. Over the years, the derivative warrant market has grown significantly. In 2012, the monthly trading value was approximately 4,000 million baht with the number of derivative warrant of 300. In 2015 the monthly trading value was approximately 40,000 million baht with the number of derivative warrant of 900. The chart of the monthly derivative warrant trading value and the number of derivative warrant in 2012 and 2015 are shown in figure1 and figure2 respectively. The number of derivative warrant has approximately increased 3 times over the past three years, while the monthly trading value has approximately increased 10 times over the past three years. The higher growth also implied the higher competition in the derivative warrant market, so it is interesting to study how the derivative warrant pricing has changed over the past years.
Figure 1. Derivative warrant trading value and the number of derivative warrant in 2012\(^1\)

![Monthly DW Trading Value vs Number of DW (Jan - Aug 2012)](image_url)

Figure 2. Derivative warrant trading value and the number of derivative warrant in 2015\(^2\)

![Monthly DW Trading Value vs Number of DW (Dec 2014 - Dec 2015)](image_url)

\(^1\) Derivative warrant trading value and the number of derivative warrant in 2012.


\(^2\) Derivative warrant trading value and the number of derivative warrant in 2015.
Derivative warrant pricing

The most original model that was used to price the option and also the derivative warrant is the Black - Scholes model. The Black - Scholes model was introduced in 1973 in a paper entitled, "The Pricing of Options and Corporate Liabilities" published in the Journal of Political Economy. The model makes certain assumptions, including:

- The options are European and can only be exercised at expiration
- No dividends are paid out during the life of the option
- Efficient markets
- No commissions
- The risk-free rate and volatility of the underlying are known and constant
- Follows a lognormal distribution which returns on the underlying are normally distributed.

The call price from the Black – Scholes model can be calculated by the following equations.

\[ C = N(d_1)S - N(d_2)Ke^{-r_cT} \]

\[ d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \ln \left( \frac{S}{K} \right) + \left( r_c + \frac{\sigma^2}{2} \right) T \right] \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]

The put price from the Black – Scholes model can be calculated by the following equations.

\[ P = N(-d_2)Ke^{-rT} - N(-d_1)S \]

- \( N(.) \) is the cumulative distribution function of the standard normal distribution
- \( T \) is the time to maturity.
- \( S \) is the spot price of the underlying asset.
- \( K \) is the strike price.
- \( r \) is the risk free rate (annual rate, expressed in terms of continuous compounding).
- \( C \) is call derivative warrant price x conversion ratio.
- \( P \) is put derivative warrant price x conversion ratio.
- \( \sigma \) is annualized standard deviation (volatility) of the continuously return on the stock.

All the variables above are known, except the volatility. Hence, the pricing of the derivative warrant only depends on the volatility which is unknown. Consequently, it has been an interesting issue to study the estimated value that the market use to represent the volatility, which is generally known as “implied volatility”.
**Motivation**

Implied volatility is the estimated volatility in the future that use to price the option and so does the derivative warrant. In the efficient market, implied volatility should be equal to the expected volatility or the subsequent realized volatility in the period that the implied volatility was expected.

However, in reality, the implied volatility is not equal to the expected volatility or the subsequent realized volatility. This evidence was also supported by the study of Limpiviriyajit (2013) which is about delta-hedged derivative warrant returns and volatility of the underlying stock by using the evidence of Thai derivative warrants. She found that Thai derivative warrant overpriced the Black-Scholes option pricing model which inferred that the implied volatility in Thai derivative warrant was higher than the subsequent realized volatility and also means the expensiveness of Thai derivative warrant.

*Figure 3. Implied volatility and Subsequent realized volatility diagram*

*Implied volatility was obtained from the derivative warrant price at time t which is predicted volatility over the maturity time for the derivative warrant. Subsequent realized volatility is*
the realized volatility covered the period between time t and the expiration date of the derivative warrant.

This also happened even in the largest option market, Chicago option market. Mixon (2009) studied the research about option market and implied volatility: Past versus present using the evidence in Chicago option market. He also found that the implied volatility in Chicago option was higher than the subsequent realized volatility and further finding is that the exceeding of the implied volatility over the subsequent realized volatility decreased from year to year and decreased significantly after the opening of the Chicago board options exchange (CBOE). Before the opening of the Chicago board options exchange (CBOE), the options were traded in over-the-counter option market. Over the years, the efficient of the market in predicting the volatility increased meaning that the “implied volatility forecast error” or the difference between the implied volatility and the subsequent realized volatility reduced. Consequently, it is an interesting issue to study whether Thai derivative warrant market which has extremely grown over these few years has the same characteristic as the Chicago option market. The first research question is to study whether the implied volatility forecast error or the difference between the implied volatility and the subsequent realized volatility reduced over the Thai derivative warrant market life.

To learn more about the implied volatility, there are many studies about how the implied volatility correlated with other factors. One of the regularities is that implied volatility tends to exceed realized volatility which inferred the high charge in pricing the derivative instruments of the issuer. This regularity was supported by the study of Black and Scholes (1972) and has been repeatedly documented in modern sample. Mixon (2009) also
tested this hypothesis and his result was also compatible with the previous study and has further finding that the exceeding of the implied volatility over the realized volatility decreased from year to year due to the higher efficient market after the CBOE opened. For Thai derivative warrant market which has dramatically grown during these recent years, the market should be more efficient due to the high competitive in the market; the derivative warrant price was expect to be cheaper, or in theoretical way is that the exceeding of the implied volatility over the realized volatility should decreased from year to year. This is the second research question.

![Figure 4. Implied volatility and Realized volatility diagram](image)

**Figure 4. Implied volatility and Realized volatility diagram**

*Implied volatility was obtained from the derivative warrant price at time t which is predicted volatility over the maturity time for the derivative warrant. Realized volatility is the realized volatility covered the period between 1-month before time t and time t.*

Another regularity of the implied volatility is that the changes in implied volatility are negatively correlated with changes in the price of the underlying stock. This regularity was supported by Schmalensee & Trippi (1978) and Sheikh (1993). In Thai derivative warrant market, the relation should be consistent and further that it is expected that the larger number
of the investors in derivative warrant market which reflects in the higher trading value over
the years would reflect the higher understanding and familiarity in derivative warrant product
meaning that there should be more investor using that derivative warrant as a hedging
instrument ,since other derivative instruments in Thailand has less liquidity comparing to the
derivative warrant.

Bollen & Whaley (2004) who studied the research whether the net buying pressure
affect the shape of implied volatility functions. He said “Daily changes in the implied
volatility of an option series are significantly related to net buying pressure ”. Consequently,
the higher demand in Thai derivative warrant over the years would also increase the effect of
the change implied volatility in put derivative warrant, while decrease the effect of the
change implied volatility in call derivative warrant. This is the last research question.
Research question

1. How does the implied volatility forecast error or the exceeding of implied volatility over the subsequent realized volatility change over history of Thai derivative warrant market? (increase or decrease)

2. How does the exceeding of implied volatility over history of Thai derivative warrant market? (increase or decrease)

3. How does the relation of implied volatility with underlying price change over history of Thai derivative warrant market?

Hypotheses

Hypothesis 1

The average implied volatility forecast error between the implied volatility and the subsequent realized volatility reduces from year to year.

Since, over the years, it is expected that the market has higher effective in predicting the volatility which would reduce the average implied volatility forecast error and bring the implied volatility to be more compatible with the Black and Scholes option pricing model. This finding supported by the study of Mixon (2009)
Hypothesis 2

The exceeding of implied volatility over the realized volatility tends to reduce from year to year.

This is because over the years, there are larger number of the derivative warrant issuers. The market should be more effective due to the higher competitive market. Consequently, it is expected that the exceeding of the implied volatility over the realized volatility should reduce to reflect the higher competition and the lower price of the derivative warrant over the years. This finding supported by the studied of Mixon (2009)

Hypothesis 3

Over the market life, the changes in implied volatility of put derivative warrant more negatively correlated with changes in the price of the underlying stock, while the changes in implied volatility of call derivative warrant less negatively correlated with changes in the price of the underlying stock.

Normally, the change in implied volatility negatively correlates with the change in underlying stock price because when the stock price decrease, the volatility increase and the implied volatility which highly correlates with the realized volatility will also increase. This finding was supported by the study of Schmalensee & Trippi (1978) and Sheikh (1993).

Since, over the years, there are larger number of the investors in derivative warrant market which reflects in the higher trading value over the years. This also reflect the higher understanding and familiarity with derivative warrant product of the investors meaning that
there should be more investor using that derivative warrant as a hedging instrument, since other derivative instruments in Thailand has less liquidity comparing to the derivative warrant. Moreover, there should be more investors trade the derivative warrant for speculative purpose because of the leverage benefit too. Bollen & Whaley (2004) said “the daily changes in the implied volatility of an option series are significantly related to net buying pressure.” Consequently, the higher demand in Thai derivative warrant in hedging purpose and speculative purpose over the years would also increase the effect of the change implied volatility.
Literature review

Limpiviriyajit (2013) studied about delta-hedge derivative warrant returns and the volatility of the underlying stock by using the evidence of Thai derivative warrants in stock exchange of Thailand. Limpiviriyajit (2013) said “The empirical results from 226 sampling derivative warrants listed in Stock Exchange of Thailand suggest that the delta-hedging by DW buyers provides a significant negative return.” According to the theory, the delta-hedge returns should be zero if the portfolio was rebalanced frequently and the implied volatility equals the realized volatility. The negative return can be implied that the implied volatility is higher than the subsequent realized volatility or means that the Thai derivative warrants overpriced the Black Scholes model.

Mixon (2009) studied about the Option markets and implied volatility: Past versus present using the evidence of Chicago options. He studied whether the modern pricing model (Black Scholes option pricing model) and centralized exchange (Chicago board option exchange) fundamentally change the way option are priced. Mixon (2009) said “Modern pricing model and centralized exchange have not fundamentally altered pricing behavior. But they have generated increased volume and a much closer conformity in the level of observed and model prices. The major change in pricing is that the implied volatility has declined relative to realized volatility over the years and the biggest portion of the decline occurred immediately after the CBOE opened.” This study found the decreasing in the different between the implied volatility and subsequent realized volatility which implied the better prediction in volatility and also found the decreasing in the different between the implied volatility and the realized volatility which implied the cheaper option price due to the higher
efficient market. This finding bring the question whether the same effect happen in Thai
derivative warrant market which has extremely grown over the past year.

Bollen & Whaley (2004) who studied the research whether the net buying pressure
affect the shape of implied volatility functions. He said “daily changes in the implied
volatility of an option series are significantly related to net buying pressure” and he found
that the prices of put options are considerably higher than is suggested by the Black–Scholes
formula and the actual level of volatility in the marketplace which was affected by the
institutional demand for portfolio insurance and the changes in implied volatility of stock
options are dominated by call option demand.

Shu & Zhang studied the relationship between implied and realized volatility of S&P
500 index. In the study, they also compared the implied volatility calculation between the
value obtained from the Black-Scholes model which is unrealistic because it assumes
constant volatility and Heston model stochastic volatility model which does not assume
volatility. The result indicated that the Heston implied volatility is more biased forecast of
realized volatility than Black-Scholes implied volatility. Consequently, the Black-Scholes
model was applied in order to calculate the implied volatility in this research.
Data

1. Derivative warrant data

Derivative warrant data was obtained from SETSMART. The period of the collected data is between 2009, the year that the derivative warrant first issued in Stock exchange of Thailand until 2015. The details of each derivative warrants was collected as followed:

- Daily closed price
- Strike price*
- Exercise ratio*
- Issuing date
- Exercise date
- Volume

* The strike price and the exercise price were obtained from 2 sources: SETSMART and Thai warrant website.

The data was used to computed the implied volatility and for screening the data.

Remark: The derivative warrant that has time to maturity less than 1 month and has no trading volume was excluded from the sample.

2. Underlying asset of each derivative warrant data

Daily close price of the underlying of each derivative warrant was collected from Bloomberg database. The period of the data is between 2008 and 2015. This data was used to compute the implied volatility, historical volatility and the weekly stock change.
3. **Risk-free rate**

1-month treasury bill was used as a proxy for risk free rate instrument because of high liquidity. The data since 2009 until 2015 was collected from Bloomberg database. The data was used to compute the implied volatility in Black-Scholes option pricing model.

4. **Market data**

Daily SET index was used as a proxy for the market data. The data since 2009 until 2015 was collected from Bloomberg database. The data was used to compute common factor ($\beta$).
Methodology

The methodology section was divided into 3 sections. The first section describes how to calculate each variable used in the test. The second section describes how to test each hypothesis in the regression model. And the last section describe the test statistic.

1. Variables

Implied volatility

The Black Scholes option pricing model was applied to calculate the implied volatility of each derivative warrant. Although, the Black Scholes option pricing model has constant volatility which is unrealistic, the study of Shu & Zhang provided evidence that the Black-Scholes model implied volatility has more explanatory power and is less biased than the Heston model (the option pricing model with stochastic volatility) implied volatility in forecasting future realized volatility. The constant risk free contrain from Black-Scholes model also does not matter because the daily risk-free rate does not change significantly.

The call price from the Black – Scholes model can be calculated by the following equations.

\[ C = N(d_1)S - N(d_2)Ke^{-rT} \]

\[ d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \ln \left( \frac{S}{K} \right) + \left( r_c + \frac{\sigma^2}{2} \right) T \right] \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]
The put price from the Black – Scholes model can be calculated by the following equations.

\[ P = N(-d_2)Ke^{-r(T)} - N(-d_1)S \]

- \( N(.) \) is the cumulative distribution function of the standard normal distribution
- \( T \) is the time to maturity.
- \( S \) is the spot price of the underlying asset.
- \( K \) is the strike price.
- \( r_c \) is the risk free rate (annual rate, expressed in terms of continuous compounding).
- \( C \) is call derivative warrant price x conversion ratio.
- \( P \) is put derivative warrant price x conversion ratio.
- \( \sigma \) is annualized standard deviation (volatility) of the continuously return on the stock.

The implied volatility was calculated and the implied volatility distribution was obtained in Appendix 1 which the implied volatility is 66.15% on the average. And the only data in range 20-110% was used as the input data in the regression while the others were considered as the outliers.

Risk free rate

Risk free rate using 1-month treasury bill has to be converted to the annualized continuous rate

\[ r_c = \ln \left(1 + \frac{r}{12}\right)^{12} \]
Realized volatility

To calculate the realized volatility, the return of the underlying has to be computed first. The return was computed by log return to be compatible for continuous time model, the Black-Scholes option pricing model

\[ r_{i,t} = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right) \]

The realized volatility was computed by the sample standard deviation over the trailing 1-month of daily data and convert to annualized volatility to be comparable with the annualized implied volatility from the Black-Scholes model.

\[ \sigma_i = \sqrt{\frac{252}{N-1}} \sqrt{\frac{\sum_{t=1}^{N}(r_{i,t} - \bar{r}_i)^2}{N-1}} \]

The subsequent realized volatility was calculated and the subsequent realized volatility distribution was obtained in Appendix2 which the implied volatility is 34.49% on the average. And the only data in range 10-80% was used as the input data in the regression while the others were considered as the outliers.

The realized volatility was calculated and the realized volatility distribution was obtained in Appendix3 which the implied volatility is 36.60% on the average. And the only data in range 10-80% was used as the input data in the regression while the others were considered as the outliers.
Beta

The beta of each underlying stock was calculated using the following equation using the data between year 2009 and year 2015.

\[ \beta_i = \frac{\sigma_i \sigma_M}{\sigma^2_M} \]

The calculated beta was contained in Appendix4.

2. Regression model

Hypothesis 1

The implied volatility forecast error between the implied volatility and the subsequent realized volatility reduces over the years.

\[
\ln(\sigma_{i,t}) - \ln(\sigma^r_{i,t+1}) = \gamma_1 + \gamma_2 d_{2010} + \gamma_3 d_{2011} + \gamma_4 d_{2012} + \gamma_5 d_{2013} + \gamma_6 d_{2014} + \gamma_7 d_{2015} + \alpha \beta_i + \theta T_{i,t} + \varepsilon_{i,t}
\]

- \( \sigma_{i,t} \) = implied volatility of derivative warrant i at time t
- \( \sigma^r_{i,t+1} \) = subsequent realized volatility of underlying stock of derivative i
- \( d_{2010} \) = dummy variable which equals one when the data was in 2010 onwards
- \( d_{2011} \) = dummy variable which equals one when the data was in 2011 onwards
• $d_{2012} = \text{dummy variable which equals one when the data was in 2012 onwards}$

• $d_{2013} = \text{dummy variable which equals one when the data was in 2013 onwards}$

• $d_{2014} = \text{dummy variable which equals one when the data was in 2014 onwards}$

• $d_{2015} = \text{dummy variable which equals one when the data was in 2015 onwards}$

• $\beta_i = \text{beta of underlying of derivative warrant i}$

• $T_{i,t} = \text{remaining lifetime derivative warrant i at time t as a percentage of the maturity of derivative warrant i at issuing date}$

If the data is in 2009, all of the dummy variables equal to zero.

In this regression, we should see each variable has the following sign if the result is consistent with the hypothesis.

• $\gamma_1$ has positive sign ($\gamma_1 > 0$). Since, the implied volatility should exceed the subsequent realized volatility supported by the studies of Limpiviriyajit (2013) using the evidence of Thai derivative warrant.

• $\gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6$ and $\gamma_7$ is negative sign ($\gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7 < 0$). Since, as time pass, the market has higher effective in predicting the volatility. This finding supported by the study of Mixon (2009) using the evidence of Chicaco options.
Hypothesis 2

The exceeding of implied volatility over the realized volatility tends to reduce over the years.

\[ \ln(\overline{\sigma_{i,t}^-}) = \alpha + \gamma_1 \ln(\overline{\sigma_{i,t}^r}) + \gamma_2 d_{2010} + \gamma_3 d_{2011} + \gamma_4 d_{2012} + \gamma_5 d_{2013} + \gamma_6 d_{2014} + \gamma_7 d_{2015} \\
+ \theta \overline{T_{i,t}} + \varepsilon_{i,t} \]

- \( \overline{\sigma_{i,t}^-} \) = time-series average implied volatility of derivative warrant i in year t
- \( \overline{\sigma_{i,t}^r} \) = time-series average realized volatility of underlying stock of derivative warrant i in year t
- \( d_{2010} \) = dummy variable which equals one when the data was in 2010 onwards
- \( d_{2011} \) = dummy variable which equals one when the data was in 2011 onwards
- \( d_{2012} \) = dummy variable which equals one when the data was in 2012 onwards
- \( d_{2013} \) = dummy variable which equals one when the data was in 2013 onwards
- \( d_{2014} \) = dummy variable which equals one when the data was in 2014 onwards
- \( d_{2015} \) = dummy variable which equals one when the data was in 2015 onwards
- \( \overline{T_{i,t}} \) = time-series average remaining lifetime derivative warrant i in year t as a percentage of the maturity of derivative warrant i at issuing date

If the data is in 2009, all of the dummy variables equal to zero.
In this regression, we should see each variable has the following sign if the result is consistent with the hypothesis.

- $\alpha$ has positive sign ($\alpha > 0$). Since, the implied volatility should exceed the realized volatility supported by the studies of Black & Scholes (1972) using the evidence of Chicaco options.

- $\gamma_2$, $\gamma_3$, $\gamma_4$, $\gamma_5$, $\gamma_6$, $\gamma_7$ is negative sign ($\gamma_2$, $\gamma_3$, $\gamma_4$, $\gamma_5$, $\gamma_6$, $\gamma_7 < 0$). Since, over the years, there are larger number of the derivative warrant issuers. The market should be more effective due to the higher competitive market. Consequently, it is expected that the exceeding of the implied volatility over the realized volatility should reduce to reflect the higher competition and the lower price of the derivative warrant from year to year. This finding supported by the studied of Mixon (2009)

**Hypothesis 3**

*Over the market life, the changes in implied volatility of put derivative warrant are more negatively correlated with changes in the price of the underlying stock, while the changes in implied volatility of call derivative warrant less negatively correlated with changes in the price of the underlying stock.*

$$
\Delta \ln(\sigma_{i,t}) = \alpha + (\gamma_1 + \gamma_2 d_{2010} + \gamma_3 d_{2011} + \gamma_4 d_{2012} + \gamma_5 d_{2013} + \gamma_6 d_{2014} + \gamma_7 d_{2015}) \Delta S_{i,t} \\
+ \partial \Delta \ln(\sigma_{M,t}) + \theta T_{i,t} + \epsilon_{i,t}
$$

Note: Since, there is no put derivative warrant issued in 2009. Consequently, the put derivative warrants was tested but using the following equation.
\[
\Delta \ln(\sigma_{i,t}) = \alpha + (\gamma_1 + \gamma_2d_{2011} + \gamma_3d_{2012} + \gamma_4d_{2013} + \gamma_5d_{2014} + \gamma_6d_{2015})\Delta S_{i,t} \\
+ \delta \Delta \ln(\sigma_{M,t}) + \theta T_{i,t} + \epsilon_{i,t}
\]

- \(\Delta \ln(\sigma_{i,t})\) = the weekly change in implied volatility in put or call derivative warrant i at time t
- \(\Delta S_{i,t}\) = the log price change of underlying stock of the derivate warrant i over a weekly interval at time t
- \(\Delta \ln(\sigma_{M,t})\) = the weekly change in realized volatility in market index (SET index) at time t
- \(d_{2010}\) = dummy variable which equals one when the data was in 2010 onwards
- \(d_{2011}\) = dummy variable which equals one when the data was in 2011 onwards
- \(d_{2012}\) = dummy variable which equals one when the data was in 2012 onwards
- \(d_{2013}\) = dummy variable which equals one when the data was in 2013 onwards
- \(d_{2014}\) = dummy variable which equals one when the data was in 2014 onwards
- \(d_{2015}\) = dummy variable which equals one when the data was in 2015 onwards
- \(T_{i,t}\) = remaining lifetime derivative warrant i at time t as a percentage of the maturity of derivative warrant i at issuing date

If the data is in 2009, all of the dummy variables equal to zero.
In this regression, we should see each variable has the following sign if the result is consistent with the hypothesis.

For the put derivative warrant, we should see the following results.

- \( \gamma_1 \) has negative sign \((\gamma_1 < 0)\). Since, the change in implied volatility are negatively correlated with the change in the price of the underlying stock. This finding was supported by the studies of Schmalensee & Trippi (1978) and Sheikh (1993)
- \( \gamma_2 , \gamma_3 , \gamma_4 , \gamma_5 , \gamma_6 \) is negative sign \((\gamma_2 , \gamma_3 , \gamma_4 , \gamma_5 , \gamma_6 < 0)\). Since, it was expected that over the years, the higher demand in derivative warrant in hedging purpose and speculative purpose would allow the issuer to charge higher price when the underlying price decrease, which reflect in the higher implied volatility change.

For the call derivative warrant, we should see the following results.

- \( \gamma_1 \) has negative sign \((\gamma_1 < 0)\). Since, the change in implied volatility are negatively correlated with the change in the price of the underlying stock. This finding was supported by the studies of Schmalensee & Trippi (1978) and Sheikh (1993)
- \( \gamma_2 , \gamma_3 , \gamma_4 , \gamma_5 , \gamma_6 , \gamma_7 \) is positive sign \((\gamma_2 , \gamma_3 , \gamma_4 , \gamma_5 , \gamma_6 , \gamma_7 > 0)\). Since, it was expected that over the years, the higher demand in derivative warrant in hedging purpose ans speculative purpose would allow the issuer to charge higher price when the underlying price increase, which reflect in the lower implied volatility change.
3. Statistic hypothesis test

The statistic hypothesis test which are applied to test the null hypothesis is the student’s t-test.

\[
t = \frac{\hat{\beta} - \beta_0}{Se(\hat{\beta})}
\]

\[
Se(\hat{\beta}) = \sqrt{\frac{1}{n-2} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2} / \sqrt{\sum_{i=1}^{n} (x - \bar{x})^2}
\]

\( \hat{\beta} \) = sample mean

\( \beta_0 \) = specified value in null hypothesis which in this test is zero

n = sample size

Degree of freedom = n-2


**Results**

**Hypothesis 1**

From the hypothesis 1, it was expected “The implied volatility forecast error between the implied volatility and the subsequent realized volatility reduces over the years.” The hypothesis was tested by using the following equation.

\[
\ln(\sigma_{i,t}) - \ln(\sigma_{i,t+1}) = \gamma_1 + \gamma_2 d_{2010} + \gamma_3 d_{2011} + \gamma_4 d_{2012} + \gamma_5 d_{2013} + \gamma_6 d_{2014} + \gamma_7 d_{2015} + \alpha \beta_i + \theta T_{i,t} + \varepsilon_{i,t}
\]

The expected results are the positive sign of the \(\gamma_1\) meaning that the implied volatility is higher than the subsequent realized volatility and the negative signs of \(\gamma_2, \gamma_3, \gamma_4, \gamma_5,\gamma_6, \text{and } \gamma_7\) meaning that the implied volatility forecast error between the implied volatility and the subsequent realized volatility reduces over the years.

The result from testing the first hypothesis contains in the table below.

**Table 1 Regression result for the hypothesis 1**

<table>
<thead>
<tr>
<th>Regression Statistics</th>
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</thead>
<tbody>
<tr>
<td>Multiple R</td>
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</tr>
<tr>
<td>R Square</td>
<td>0.143611</td>
</tr>
<tr>
<td>Adjusted R Square</td>
<td>0.143586</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.297583</td>
</tr>
<tr>
<td>Observations</td>
<td>271903</td>
</tr>
</tbody>
</table>
From the above table, the significant positive sign of the intercept coefficients confirms the study of Limpiviriyajit (2013) that the implied volatility in Thai derivative
warrant is higher than the subsequent realized volatility meaning that the Thai derivative warrant market overestimates the volatility.

However, the implied volatility forecast error of Thai derivative warrant does not reduce as it was expected in the hypothesis. This can be seen from the coefficients of the \( d_{2010}, d_{2011}, d_{2012}, d_{2013}, d_{2014}, d_{2015} \) which are significant but not all negative. Even in the long run the implied volatility forecast error still does not reduce. This can be seen from the sum of the dummy variable coefficients \( (d_{2010}, d_{2011}, d_{2012}, d_{2013}, d_{2014}, d_{2015}) \) which is still positive value. Since the implied volatility forecast error does not reduce over the year, it means that Thai derivative warrant market does not have higher effective in predicting the volatility over the years.

Moreover, it was found the reversion of the implied volatility forecast error from year to year which can be seen from the switching sign of the coefficients of the dummy variables. The following graph shows the reversion of the implied volatility forecast error over the years.

\[
\text{Average of } \ln(\text{Implied volatility}) - \ln(\text{Subsequent realized volatility})
\]

\[ 9/7/2009 \quad 9/7/2010 \quad 9/7/2011 \quad 9/7/2012 \quad 9/7/2013 \quad 9/7/2014 \quad 9/7/2015 \]

*Figure 3. The implied volatility forecast error from year 2009 to year 2015*
To analyze more for the reversion of the implied volatility forecast error, each elements of the implied volatility was studied: the implied volatility and the subsequent realized volatility. The following graph shows the movement of the implied volatility and the movement of the subsequent realized volatility.

![Graph showing implied volatility and subsequent realized volatility from year 2009 to year 2015.](image)

**Figure 4. The implied volatility and the subsequent realized volatility**

*from year 2009 to year 2015*

In the above graph, it can be seen that from year 2011 to year 2015, the implied volatility is quite smooth, while the subsequent realized volatility moves significantly over the years. Consequently, this can be conclude that the reversion of the implied volatility forecast error is the results of the subsequent realized volatility. The movement of implied volatility and the subsequent realized volatility of each derivative warrant also studied to perform the consistency of this conclusion. The graphs of the representative derivative warrant were shown in Appendix 5.
Note: The peak of the implied volatility in September 2010 – November 2010 was affected by the large demand of the Thai derivative warrant in that blooming period of Thai derivative warrant. The graph of the trading volume of that period was shown in Appendix6.

Hypothesis 2

From the hypothesis 2, it was expected “The exceeding of average implied volatility over the average realized volatility tends to reduce over the years.” The hypothesis was tested by using the following equation.

\[
\ln(\frac{\sigma_i}{\sigma \bar{t}}) = \alpha + \gamma_1 \ln(\frac{\sigma_i}{\sigma \bar{t}}) + \gamma_2 d_{2010} + \gamma_3 d_{2011} + \gamma_4 d_{2012} + \gamma_5 d_{2013} + \gamma_6 d_{2014} + \gamma_7 d_{2015} + \theta T_{i,t} + \epsilon_{i,t}
\]

The expected results are the positive sign of the \( \alpha \) meaning that the implied volatility is higher than the realized volatility and the negative signs of \( \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7 \) meaning that the exceeding of implied volatility over the realized volatility tends to reduce over the years. The result for testing the first hypothesis contains in the table below.
Table 2 Regression result for the hypothesis 2

<table>
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<tbody>
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<tr>
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<thead>
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<td>(SS)</td>
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<tr>
<td>Regression</td>
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<td>Residual</td>
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<tr>
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<th>P-value</th>
<th>Lower 95%</th>
<th>Upper 95%</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.093229</td>
<td>25.83317</td>
<td>8.3292E-140</td>
<td>2.225643309</td>
</tr>
<tr>
<td>\ln(\sigma_{i,t}^2)</td>
<td>0.463856</td>
<td>0.006673</td>
<td>69.51363</td>
<td>0</td>
<td>0.450774598</td>
</tr>
<tr>
<td>d_{2010}</td>
<td>0.340947</td>
<td>0.092626</td>
<td>3.680887</td>
<td>0.000234411</td>
<td>0.159367256</td>
</tr>
<tr>
<td>d_{2011}</td>
<td>-0.15683</td>
<td>0.023731</td>
<td>-6.60871</td>
<td>4.2019E-11</td>
<td>-0.203351651</td>
</tr>
<tr>
<td>d_{2012}</td>
<td>0.112713</td>
<td>0.010041</td>
<td>11.22496</td>
<td>5.86473E-29</td>
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</tr>
<tr>
<td>d_{2013}</td>
<td>-0.08134</td>
<td>0.007603</td>
<td>-10.6991</td>
<td>1.75395E-26</td>
<td>-0.096245506</td>
</tr>
<tr>
<td>d_{2014}</td>
<td>0.078684</td>
<td>0.006238</td>
<td>12.61311</td>
<td>4.96918E-36</td>
<td>0.066454673</td>
</tr>
<tr>
<td>d_{2015}</td>
<td>-0.1102</td>
<td>0.005016</td>
<td>-21.9687</td>
<td>4.7541E-103</td>
<td>-0.120037357</td>
</tr>
<tr>
<td>\bar{T}_{i,t}</td>
<td>-0.11366</td>
<td>0.010011</td>
<td>-11.3535</td>
<td>1.39981E-29</td>
<td>-0.133289427</td>
</tr>
</tbody>
</table>

The significant positive sign of the intercept coefficients means that the average implied volatility in Thai derivative warrant exceeds the average realized volatility which is consistent with the study of Black & Scholes (1972).

However, the exceeding of average implied volatility over the average realized volatility does not reduce over the years as it was expected in the hypothesis 2. This can be seen from the signs of the d_{2010}, d_{2011}, d_{2012}, d_{2013}, d_{2014}, d_{2015} coefficients which are significant but not all negative. This means that even there is larger number of the derivative warrant issuers. This higher competitive market does not drive the exceeding of the implied volatility over the realized volatility to reduce or bring the lower price of the derivative warrant from year to year. Consequently, the higher competitive in Thai derivative warrant
market does not drive Thai derivative warrant market to be more efficient and the cheaper price of Thai derivative warrant.

Moreover, it was found the reversion of the exceeding of the implied volatility over the realized volatility from the switching sign of coefficients of the dummy variable. However, from the further study of each element over the years. It seems like the reversion of the exceeding over the years was affected by the movement of the average realized volatility. The following graph shows the time-series average of the implied volatility and the realized volatility each year.

![Average of implied volatility and Average of Realized volatility](image.png)

*Figure 4. The time-series average of the implied volatility each year and the time-series average of the subsequent realized volatility each year from year 2009 to year 2015*

From the analysis of each derivative warrant, it was found unclear pattern that the reversion of the exceeding of the implied volatility over the realized volatility caused by which element. The graphs of the time-series average of the implied volatility each year and the time-series average of the subsequent realized volatility each year of each derivative
warrant was shown in Appendix7. However, it can be only concluded that the higher competitive of the Thai derivative warrant over the years drive neither the more effective market, nor the cheaper price of the Thai derivative warrant.

**Hypothesis 3**

From the hypothesis 3.1 (for the put derivative warrants), it was expected “Over the market life, the changes in implied volatility of put derivative warrant more negatively correlated with changes in the price of the underlying stock.” The hypothesis was tested by using the following equation.

\[
\Delta \ln(\sigma_{t,t}) = \alpha + (\gamma_1 + \gamma_2 d_{2011} + \gamma_3 d_{2012} + \gamma_4 d_{2013} + \gamma_5 d_{2014} + \gamma_6 d_{2015}) \Delta S_{t,t} \\
+ \partial \Delta \ln(\sigma_{M,t}) + \theta T_{i,t} + \varepsilon_{i,t}
\]

The expected results are the negative signs of the $\gamma_1$, $\gamma_2$, $\gamma_3$, $\gamma_4$, $\gamma_5$, $\gamma_6$ coefficients. Since, it was expected that over the years, the higher demand in derivative warrant in hedging purpose would allow the issuer to charge higher price when the underlying price decrease, which reflect in the higher implied volatility change.
Table 3 Regression result for the hypothesis 3 (Put derivative warrant data)

<table>
<thead>
<tr>
<th>Regression Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
</tr>
<tr>
<td>R Square</td>
</tr>
<tr>
<td>Adjusted R Square</td>
</tr>
<tr>
<td>Standard Error</td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ANOVA</th>
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</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>df</td>
</tr>
<tr>
<td>Regression</td>
</tr>
<tr>
<td>Residual</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
<th>Lower 95%</th>
<th>Upper 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.00615</td>
<td>0.000858</td>
<td>-7.16316</td>
<td>7.99E-13</td>
<td>-0.00783</td>
<td>-0.00447</td>
</tr>
<tr>
<td>ΔS_{i,t}</td>
<td>0.765493</td>
<td>0.244591</td>
<td>3.12969</td>
<td>0.001751</td>
<td>0.286092</td>
<td>1.244893</td>
</tr>
<tr>
<td>d_{2011}ΔS_{i,t}</td>
<td>-0.58727</td>
<td>0.245817</td>
<td>-2.38907</td>
<td>0.016895</td>
<td>-1.06908</td>
<td>-0.10547</td>
</tr>
<tr>
<td>d_{2012}ΔS_{i,t}</td>
<td>0.006944</td>
<td>0.030511</td>
<td>0.227594</td>
<td>0.819963</td>
<td>-0.05286</td>
<td>0.066746</td>
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<tr>
<td>d_{2013}ΔS_{i,t}</td>
<td>-0.13003</td>
<td>0.020124</td>
<td>-6.46138</td>
<td>1.05E-10</td>
<td>-0.16947</td>
<td>-0.09059</td>
</tr>
<tr>
<td>d_{2014}ΔS_{i,t}</td>
<td>-0.01076</td>
<td>0.013781</td>
<td>-0.78099</td>
<td>0.434809</td>
<td>-0.03778</td>
<td>0.016249</td>
</tr>
<tr>
<td>d_{2015}ΔS_{i,t}</td>
<td>0.117676</td>
<td>0.01679</td>
<td>7.008629</td>
<td>2.44E-12</td>
<td>0.084767</td>
<td>0.150585</td>
</tr>
<tr>
<td>Δln(σ_{M,t})</td>
<td>-0.03299</td>
<td>0.006515</td>
<td>-5.06284</td>
<td>4.15E-07</td>
<td>-0.04576</td>
<td>-0.02022</td>
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<tr>
<td>T_{i,t}</td>
<td>0.00534</td>
<td>0.001343</td>
<td>3.975339</td>
<td>7.04E-05</td>
<td>0.002707</td>
<td>0.007972</td>
</tr>
</tbody>
</table>

The results was really surprised that the sign of ΔS_{i,t} coefficient is significant and positive meaning that when the stock price decrease, the implied volatility also decrease which contrast to the fact that when the stock price decreases, the realized volatility increases and leads the implied volatility which positively correlated to the realized volatility to increase. Even there are some negative signs of the d_{2011}ΔS_{i,t}, d_{2012}ΔS_{i,t}, d_{2013}ΔS_{i,t}, d_{2014}ΔS_{i,t}, d_{2015}ΔS_{i,t} coefficients, they are not large enough to offset the positive value of the ΔS_{i,t} coefficient meaning that even from year to year, the movement of the stock price still positively correlate to the movement of the implied volatility. Consequently, it can be concluded that the implied volatility in Thai derivative warrant market has not affected by the
the higher demand of the put derivative warrant and the put derivative warrant is considered cheaper when the stock price decreases due to the lower implied volatility.

From the hypothesis 3.2 (for the call derivative warrants), it was expected “Over the market life, the changes in implied volatility of call derivative warrant less negatively correlated with changes in the price of the underlying stock.” The hypothesis was tested by using the following equation.

\[
\Delta \ln(\sigma_{i,t}) = \alpha + (\gamma_1 + \gamma_2 d_{2011} + \gamma_3 d_{2012} + \gamma_4 d_{2013} + \gamma_5 d_{2014} + \gamma_6 d_{2015}) \Delta S_{i,t} \\
+ \partial \Delta \ln(\sigma_{M,t}) + \theta T_{i,t} + \epsilon_{i,t}
\]

The expected results are the negative signs of the \( \gamma_1 \) and the positive signs of the \( \gamma_2 \), \( \gamma_3 \), \( \gamma_4 \), \( \gamma_5 \), \( \gamma_6 \) coefficients. Since, it was expected that over the years, the higher demand in derivative warrant in hedging purpose and speculative purpose would allow the issuer to charge higher price when the underlying price increase, which reflect in the lower implied volatility change.
Table 4 Regression result for the hypothesis 3 (Call derivative warrant data)

<table>
<thead>
<tr>
<th>Regression Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
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<tr>
<td>R Square</td>
</tr>
<tr>
<td>Adjusted R Square</td>
</tr>
<tr>
<td>Standard Error</td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

ANOVA

<table>
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<th>MS</th>
<th>F</th>
<th>F</th>
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<td>1.906876</td>
<td>239.4855</td>
</tr>
<tr>
<td>Residual</td>
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<td>1215.928</td>
<td>0.007962</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>152718</td>
<td>1233.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Coefficients</td>
<td>Standard Error</td>
<td>t Stat</td>
<td>P-value</td>
</tr>
<tr>
<td>-----------------------</td>
<td>--------------</td>
<td>----------------</td>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.00457</td>
<td>0.00064</td>
<td>-7.13991</td>
<td>9.38E-13</td>
</tr>
<tr>
<td>$\Delta S_{i,t}$</td>
<td>-2.09941</td>
<td>0.151479</td>
<td>-13.8595</td>
<td>1.19E-43</td>
</tr>
<tr>
<td>$d_{2010}\Delta S_{i,t}$</td>
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<td>0.159439</td>
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<td>$d_{2011}\Delta S_{i,t}$</td>
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<td>8.489638</td>
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<td>$d_{2013}\Delta S_{i,t}$</td>
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<td>0.000101</td>
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<tr>
<td>$d_{2014}\Delta S_{i,t}$</td>
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<tr>
<td>$d_{2015}\Delta S_{i,t}$</td>
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<td>7.7E-14</td>
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<tr>
<td>$\Delta \ln(\sigma_{M,t})$</td>
<td>0.009325</td>
<td>0.005236</td>
<td>1.781027</td>
<td>0.07491</td>
</tr>
<tr>
<td>$T_{i,t}$</td>
<td>0.003626</td>
<td>0.000989</td>
<td>3.665547</td>
<td>0.000247</td>
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</table>

The results absolutely differ from the put derivative warrant data. The movement of implied volatility of the call derivative warrant negatively correlated with the movement of the stock price which consistent with the many studies. This can be seen from the significant and negative sign of the $\Delta S_{i,t}$ coefficient. Moreover, it was found that the coefficients of $d_{2010}\Delta S_{i,t}$, $d_{2011}\Delta S_{i,t}$, $d_{2012}\Delta S_{i,t}$, $d_{2013}\Delta S_{i,t}$, $d_{2014}\Delta S_{i,t}$ are significant positive meaning that from year to year the changes in implied volatility of call derivative warrant less negatively correlates with changes in the price of the underlying stock. Alternatively, it means that when the stock price increase, the implied volatility decrease which can be found from the negative sign of the $\Delta S_{i,t}$ coefficient, but from year to year, the negative amount is lower because
there is more hedging and speculative demand for the call derivative warrant when the stock price increase. This can be conclude that over the years in year 2009-2014, there is higher demand for the call derivative warrant decrease the effect of the negative change in implied volatility when the stock price change in the same amount.
Conclusion

Thai derivative warrants is overvalue comparing to the Black-Scholes pricing theory which can be implied that the prediction of the volatility is overestimated. Over the years, the prediction of the volatility is not better which can be seen from the implied volatility forecast error that does not reduce over ther years. However, it was found the reversion of the implied volatility forecast error which is the result from the movement of the subsequent realized volatility over the years.

It was also found that even there is higher competition in Thai derivative warrant which comes from the higher number of the issuers over the years, the Thai derivative warrant is not cheaper which can be seen from the exceeding of the time-series average of the implied volatility over the time-series average of the realized volatility that does not reduce over the years. The reversion of the exceeding also found but cannot be comclude whether it comes from which element.

The demand also affects the pricing in the derivative warrant. The higher the demand over the years increase the call derivative warrant price when the stock price increases which can be seen from the lower negative correlation of the change in implied volatility and the change in stock price. However, this effect does not happen with the put derivative warrant pricing. And the surprising is that the change in put derivative warrant implied volatility positively correlated the change in stock price meaning that the put derivative warrant implied volatility is lower when the stock price decreases.
Bibliography


*Student's t-test*. (2016, 1 30). Retrieved from Wikipedia Web Site:

https://en.wikipedia.org/wiki/Student%27s_t-test
Appendix

Appendix 1. The implied volatility distribution

The average annualized implied volatility is 66.15%.

Appendix 2. The subsequent realized volatility distribution

The average annualized subsequent realized volatility is 34.49%.
Appendix 3. The realized volatility distribution

The average annualized realized volatility is 36.60%.
## Appendix 4. Beta of the underlying

<table>
<thead>
<tr>
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<th>Beta</th>
<th>Underlying</th>
<th>Beta</th>
<th>Underlying</th>
<th>Beta</th>
</tr>
</thead>
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<td>0.870578</td>
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<td>GLOW</td>
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<td>S</td>
<td>0.971713</td>
</tr>
<tr>
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<td>GUNKUL</td>
<td>1.032273</td>
<td>S50</td>
<td>1.097797</td>
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<td>ANAN</td>
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</tr>
<tr>
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<td>SCB</td>
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<td>0.862886</td>
<td>SCC</td>
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<tr>
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<td>0.721096</td>
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Appendix 5

The movement of the implied volatility and the subsequent realized volatility

Appendix 5.1 The average implied volatility and the average subsequent realized volatility of the derivative warrants that have PTT as the underlying stock

Appendix 5.2 The average implied volatility and the average subsequent realized volatility of the derivative warrants that have SCC as the underlying stock
Appendix 5.3 The average implied volatility and the average subsequent realized volatility of the derivative warrants that have KBANK as the underlying stock.

Appendix 5.4 The average implied volatility and the average subsequent realized volatility of the derivative warrants that have SCB as the underlying stock.
Appendix 6

The trading volume of Thai derivative warrant in September 2010 - November 2010

Appendix 6.1 The trading volume of TTA13CA in September 2010 - November 2010

Appendix 6.2 The trading volume of PS13CA in September 2010 - November 2010
Appendix 6.3 The trading volume of PTTC13CA in September 2010 - November 2010

Appendix 6.4 The trading volume of PTTE13CB in September 2010 - November 2010
Appendix 6.5 The trading volume of KBAN13CB in September 2010 - November 2010

Appendix 6.6 The trading volume of SCB08CA in September 2010 - November 2010
Appendix 6.7 The trading volume of PTT13CC in September 2010 - November 2010

Appendix 6.8 The trading volume of IVL01PA in September 2010 - November 2010
Appendix 6.9 The trading volume of BANP13CA in September 2010 - November 2010

Appendix 6.10 The trading volume of CPF08CA in September 2010 - November 2010
Appendix 6.11 The trading volume of BANP13CB in September 2010 - November 2010

Appendix 6.12 The trading volume of ADVA13CA in September 2010 - November 2010
Appendix 7

The time-series average of the implied volatility each year and the time-series average of the subsequent realized volatility each year of each derivative warrant

Appendix 7.1 The time-series average of the implied volatility each year and the time-series average of the subsequent realized volatility each year of the derivative warrants that have KBANK as the underlying stock
Appendix 7.2 The time-series average of the implied volatility each year and the time-series average of the subsequent realized volatility each year of the derivative warrants that have ADVANC as the underlying stock.

Appendix 7.3 The time-series average of the implied volatility each year and the time-series average of the subsequent realized volatility each year of the derivative warrants that have SCB as the underlying stock.
Appendix 7.4 The time-series average of the implied volatility each year and the time-series average of the subsequent realized volatility each year of the derivative warrants that have PTT as the underlying stock

Appendix 7.5 The time-series average of the implied volatility each year and the time-series average of the subsequent realized volatility each year of the derivative warrants that have IVL as the underlying stock
Appendix 7.6 The time-series average of the implied volatility each year and the time-series average of the subsequent realized volatility each year of the derivative warrants that have BAA/NPU as the underlying stock.